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ABSTRACT

Presented in this document are essays related to a research project focusing on conceptual obstacles which confine and inhibit scientific understanding (conceptual barriers). These include a short summary of research activities; research methodology; outline of a theory and summary of supporting evidence related to conceptual barriers encountered in teaching science to adults; students' views on critical barriers; a taxonomy of critical barriers (focusing on pervasive barriers, barriers recognized in narrower contexts, and barriers related to mathematics); problems with mathematics; ecological contexts of critical barriers (psychological sources of learning problems, differences in personal learning styles, and problems with pedagogenic causes); clues from science history (biographical/historical sources of critical barriers); description of a college level "Man and Nature-Energy" course, including list of difficulties students encountered; and notes and reflections on the course by a participant/observer. A list of references, annotated list of relevant psychological literature, and the proposal submitted to the National Science Foundation (NSF) are included. The NSF proposal provides the rationale/background for and significance of the research. The document is associated with five additional volumes of detailed records. Contents of these volumes are included at the beginning of this document. (Author/JN)

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A REPORT OF

RESEARCH ON CRITICAL BARRIERS TO THE LEARNING AND UNDERSTANDING OF ELEMENTARY SCIENCE

National Science Foundation Contract # SED 80-08581

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Preface

This volume of essays, associated with five additional volumes of detailed records, represents a partial culmination of two years of research on science teaching. In these essays we have undertaken to abstract, analyze, and give theoretical formulation to material contained in the record (and, of course, in personal notes and memories).

The record is one of teaching sessions in two types of courses, one for undergraduates and one for elementary school teachers, inservice; it is also a record of teachers' commentaries and of our staff discussions along the way, plus other material which seemed appropriate to include (Table of Contents, below).

In our original proposal (Appendix B) we asked for financial support over a three year period. As finally granted our support from the National Science Foundation was for half of that period, with the injunction that we were to demonstrate the feasibility of the procedures we had proposed to use. In the matter of time we have found sufficient non-NSF funding so that we could continue our work for an extra semester, and augmenting our staff for the last year by one half-time researcher.

Our major activities, as mentioned above, have been two intensive teaching-observing commitments. One of these was to co-teach and observe an undergraduate course, Man and Energy, in the Fall of 1980 and again in the Spring of 1981. The description of this course is contained in Ron Colton's paper on the Energy course (pp. 1) and Maja Apelman's paper (pp. 1) and a commentary on it in the essay, Notes and Reflections on Course by Maja Apelman. Dr. Apelman, a long-time member of the staff of the Mountain View Center, experienced as a teacher of young children and as a professional advisor to elementary school teachers, describes herself as scientifically naive, encountering conceptual difficulties in science which are often on a par with, and tuned to, those of the students she is observing. This circumstance is at once a liability and an asset. As a reading of her essay (first reference) will show, the asset will often be that she anticipates and understands, in her own person, the very difficulties which students are experiencing.

Ronald Colton took part in this course both semesters, I (David Hawkins) in one. We each gave occasional lectures, and shared a laboratory/discussion section.

The outstanding opportunity provided by participation in this course was the opportunity to observe students' struggles with our joint efforts to expand what can be called their horizons of scale: to deal simply, but meaningfully and with some sense for reliable approximation, with planetary affairs but also, at the same time, with atoms and nuclei. Much of our experience in this course is reflected in Professor Ronald Colton's two essays, "Problems of Mathematics" and "Man and Nature - Energy Courses".

This major commitment of time was valuable to us in several ways, explicated in the essays mentioned above. We would not again make such a commitment, however, without a substantial alteration in either the plan of the course or in our own research aims.

As matters actually developed the course became too demanding of time for preparation, teaching and tutoring, affording almost no time for leisurely class discussion or research interviews.

The major source of evidence concerning critical barriers and related matters is derived, in consequence, from our two semester-long courses for teachers. A detailed record of these courses occupies the greater part of five volumes (see Contents below). This record includes almost all of our little lectures and discussions. It excludes periods when we were in small groups for lab work, and a few other occasions. On these occasions we kept individual notes not reproduced here.

The teachers in these two courses (about 10 in each group) were selected on the basis of their responses to descriptions which we circulated. Several of them were already well known to us from their previous association with the Mountain View Center, and known as able and serious professional teachers. They could also be described, mainly, as scientifically naive, for the most part lacking even the rudiments of modern understanding in the physical and natural sciences. Finally, and most importantly, these teachers came forward with some understanding of our purpose. This purpose was the dual one of helping them learn more elementary science, relevant to their own teaching, and of enlisting their help in describing and accounting for the conceptual difficulties they had already experienced in learning science, or would experience under our tutelage.

The topics for the first semester were Size and Scale, and Heat and Temperature. For the second semester the sole topic was Light and Color.

The essays included in this volume are diverse in style and viewpoint. This diversity reflects the stage to which our research has advanced, a stage at which there is an embarassment of riches, a record containing far more than we have been willing, or able, to use. For such data as can be extracted from our five volumes of record there is no easy summary possible, and for most readers our samplings from them, in these essays, must be taken on faith. The original volumes are of course available for scrutiny by anyone sufficiently interested. Parts of them would prove fascinating reading, while other parts (poorly transcribed, for example) would prove almost unintelligible to one not cued in by personal memory.

In another year, as originally requested, we could have sampled more widely across the range of ages, backgrounds, and subject matter. The relation of childrens' conceptual difficulties to those of adults goes unexamined, for example, moreover we have not been able to conduct the right kind of search for barrier phenomena in the biological and social sciences, a challenge we had hoped to meet.

In another year we could also have arrived at a more unified account of the several topics reported here. Having duly recorded this apologia, however, we would like to express our conviction that we have indeed helped define a major area of educational search and research, and by positive findings in this area have demonstrated the feasibility of a style of investigation very close to the normal work of the classroom, though one slowed down from a conventional tempo and enriched by the luxury of adequate staff and adequate time for reading the record and planning its extension. This style of investigation can fail in obvious ways to meet the canons of a strict methodology. If however it directs attention to important phenomena, and does so with sufficient redundancy and persuasiveness, the failings can be rather easily remedied.

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Fall 1980

A college course for non-science majors offered by the University's Center for Interdisciplinary Studies in partial fulfillment of the College of Arts and Sciences' natural science requirement.

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SUMMARY
RESEARCH ACTIVITIES

FALL 1980

1. College Course - Man & Nature: Energy

An undergraduate college course for non-science majors which fulfilled the natural science requirements of students in the University's College of Arts and Sciences had been selected for our research in the fall of 1980. Man and Nature: Energy was team taught by three instructors, one of whom, Ron Colton, was a member of our research staff. The other two instructors, David Armstrong and Benno Klank were interested in our research questions and volunteered to participate as much as time allowed. Maja Apelman attended all the lectures of the course and acted as participant observer in many of the lab/discussion sessions led by Ron Colton and Benno Klank. Her essay Notes and Reflections on Course: Man and Nature: Energy describes her experiences as participant observer in this class. Additional material on this class can be found in Volume I, pp. 125-248.

2. Faculty Seminar

During the fall semester, the faculty seminar group met twice. Jack Easley from the University of Illinois led one of the meetings and also consulted with the research staff. (Transcripts of these meetings can be found in Volume I.)

SPRING 1981

1. The course Man and Nature: Energy was repeated in the spring semester. Ron Colton and David Hawkins co-taught one of the lab/discussion sections and also spent some extra time with individual students who had a particularly hard time with the mathematics of the class.

2. Teachers' Seminar I

Teachers who had had previous contact with the Mountain View Center as well as teachers who had never taken any courses at the Center were invited to become research participants. The seminar met once a week for two hours after school for a period of fifteen weeks. Two topics were studied: Size and Scale and Heat. For brief summaries of the class content, see Volume III, pp. 2-3. Volumes II and III contain the data collected in this seminar.

3. Faculty Seminar

There were two more meetings of the faculty seminar, one of them led by Ken Hammond of the University's Psychology Department. (See Volume I for transcripts.)

JUNE 1981

Evaluation and Planning

Abe Flexer joined our research team and we had a series of evaluation and planning sessions in which we tried to decide on both the format and the content for the second teachers' seminar. Partly because of teachers' stated interests (see Interviews with Teachers, Volume II, pp. 168-178) and partly because the staff thought that it would be a fruitful topic, we decided to study Light and Color. After much discussion about other possible formats, we agreed to stay with the weekly 2-hour after-school sessions, adding a full day Saturday class as well as two individual or small group sessions for each student at which they could get more attention with problems and which might give the staff further insight into the nature of critical barriers.

FALL 1981

1. Teachers' Seminar No. 2 on Light and Color

Summaries of individual classes can be found in Volumes IV and V, pp.2-2a.

The data from this seminar can be found in Volumes IV and V.

2. Class for Undergraduates in Teacher Education

David Hawkins and Ron Colton taught a special class for a small group of undergraduates in teacher education who were interested both in learning science and in doing science projects with groups of school children. Ann Drucker, who reported a discussion on white light by a group of children (see David Hawkins' essay, p.   ) was a student in this class.

SPRING 1982

Evaluating, Organizing, Writing

For many weeks, the research staff met weekly, going over the data that had been collected, reading related research literature, searching for organizing themes and finally arriving at a way in which our findings could best be presented. The remainder of the spring semester was spent in writing and putting together the research report.

OUTLINE RESEARCH METHODOLOGY

Introduction: The Nature of the Task

David Hawkins, in his Introductory Statement, has pointed out the naturalistic, natural history character of our research, and likened it to the *Challenger* expedition of the last century which was the first voyage of oceanographic exploration to add significantly to our knowledge of the biology of the oceans, and in particular of the sorts of creatures which inhabit it. It is perhaps worth pursuing this analogy a little further in the hope it will clarify the nature of our mission to search for difficulties rather than to research into them.

For long ages before the development of marine biology as a science, mankind had been aware of the richness of life in the ocean, mainly through fisherman. They kept what they knew to be useful and threw the rest overboard. Some organisms had been studied in great detail, but vast areas were unexplored. Organisms that were known were classified, but anything approaching a complete picture was impossible with so limited a search. What was needed was a systematic search over a chosen but fairly extensive area to find out what sorts of creatures were there. More detailed studies of these organisms and their relationships could then be carried out, while at the same time other areas could be explored.

We felt ourselves in a position analogous to that of the Challenger staff. Practising teachers at all levels had noticed difficulties, some of which occurred frequently. A few had begun to classify those they had found (for example, McDermott*). Others had investigated individual problems in considerable depth. But a careful search to uncover difficulties was missing. This was our commission. We simply went fishing.

---

* Lillian C. McDermott, Leonie K. Piternick, and Mark L. Rosenquist, "Helping Minority Students Succeed in Science: I. Development of a Curriculum in Physics and Biology", Journal of College Science Teaching, January 1980, pp. 136-138.

The various stages of exploration and investigation are not necessarily sequential; dissection, study, and classification of what one has already found proceed while further explorations are underway. Finally, whether it be in the real oceans or in the ocean of human learning, at least some of what we have found can be put to use. To understand the obstructions we encountered, we tried to find our way around them; that is, where the pathway to learning we had planned ran into difficulties, we had to explore alternative pathways; we had to imporve our teaching. So we hope that our studies may be of use to curriculum developers in sensitizing them to some of the barriers that exist, and perhaps in suggesting some ways in which those barriers have been built up, and steps which individuals have had to climb to overcome them.

The Research Situation

Piaget, whose work we value, has been concerned with the development of human understanding from infancy through adolescence. He has always seen this development as in essence unaffected by formal education.

We, on the contrary, have been interested in the conflict between the thought habits involved in scientific understanding and exploration on the one hand, and the habits of thought available to persons who have little or no scientific understanding (our term is "scientifically naive"). On the other hand it should be added, also, that Piaget never studied adults.

Since we were concerned with a search for difficulties that arise in the course of learning science, our vehicle had to be a teaching situation in which we could observe the students' responses to the material we presented to them and to our style of teaching. We had to create an interactive situation, a to and fro between teacher and taught, in which the teaching would be immediately responsive to the reactions of the learners, eliciting feedback, probing for alternative explanations to find ways round difficulties, and so probing ever deeper into the nature of the difficulties themselves.

Other methods, we felt, either restricted the range of material or inhibited the freedom of response.

Interviews must focus in a narrow front in which the interviewer already knows what he wants to discuss. They seem suited to deeper probing into barriers that have already appeared, rather than to the initial search. The formal questionnaire or test is even more inflexible, and appeared quite unsuitable to either the broad sweep of our search or to the deeper probing of difficulties, often unsuspected, as they appeared.

The style of our teaching would involve:

a) A minimun of didactic teaching.

b) Small numbers so that each student would be treated as an individual.

c) Plenty of opportunity for practical work to enable the students to confront the various phenomena.

d) A pace determined by the interests, progress, and needs of the students and not by some predetermined schedule.

e) Ample opportunity for discussion, to allow students to voice their difficulties. (This turned out to be even more important than we had expected because the interchange in these discussions enabled students to clarify and express their difficulties more clearly. One person's statement would remind others of points they had forgotten or perhaps were a little reluctant to express. Furthermore, the flow of discussion, exposing different facets of the topic and varying problems, enabled the researchers to interject fresh insights, alternative ways of clarifying the matter, new probings into difficulties which greatly enriched our data.

f) A team-teaching situation to allow for individual attention, divergent approaches, and detailed observation.

g) Above all, an atmosphere of confidence and trust in which the students would feel themselves to be collaborators and not guinea pigs, and in which they would air their difficulties without feeling threatened; in which the simplest problem would be respected, not as a dumb question but as an attempt to probe ever more deeply into the process of learning.* (This cannot be emphasized too much. Those involved had to have complete trust in the research team and each other if difficulties were to be freely expressed and discussed.

h) Furthermore, we had to allow and indeed encourage difficulties and frustrations to appear, and yet make the overall course a worthwhile and satisfying experience for the participants.

i) The teaching style had to be one which opens up possibilities for exploration of the phenomena being dealt with and that allows students to take their time, to follow promising clues, and to backtrack when this seems profitable for learning. The teacher needs to be sensitive to feedback from the students and must be prepared to invent alternative pathways to understanding when difficulties arise. Above all, the teacher must be prepared to follow the often-quoted, seldom-followed precept that the root of the word "educate" means "to draw out", so that the process is not that of leader and follower but a joint probing into different ways of understanding the world and a gradual reconciliation of individual intuitive views with accepted scientific explnations.

WHOM Should We Teach

The research team had experience in teaching all levels from pre-school to graduates and adult education classes, including initial and in-service training of teachers. We also had experience with Oglalla Sioux and Hopi teachers and children. Had the project been funded for the full period of the proposal, we would have become involved with all of these groups. With the limited time and resources available, we confined ourselves to

(i) university non-science majors, because we had access to a class of 80 students who would be team-taught by faculty sharing our interests, whose style of teaching, reflecting the needs of individuals, would allow us to identify individual difficulties.

(ii) Elementary school teachers, who would in the main be scientifically naive, who would be interested in the processes of learning and the difficulties encountered, and who, because of previous work with us, would be at ease at the Center and would not feel in any way threatened in revealing their own learning problems. They would also be interested in carrying forward the investigation with their own classes.

In chosing adults for our subjects we were freeing ourselves also from any rigid connection with age-linked Piagetian stages.

The Class as a Medium for Our Research

While it became apparent during the first semester that this class was not ideal for our purposes, it seemed sufficiently fruitful for us to persist for another semester. More difficulties were uncovered, but the principal return was a very substantial confirmation of the difficulties that had already arisen. However, in light of our experience of the teachers' courses discussed in the papers listed on pages 10-12 of these notes.

it is now clear to us that these gave a much better return for the effort involved. The clear advantages to be gained from a small group of selected subjects, who quickly became completely at ease, who had no examination to face and so nothing to lose, and who had a professional interest in the process of learning, are described in that paper. Above all, the fact that we were exploring a topic rather than offering a course meant that we were free to modify the style, content, and pace of the class to suit the needs of individuals in a way that would have been quite inappropriate in a University course offered for credit which had to meet certain requirements of the College of Arts and Sciences.

1. Size of Class

Even the lab sections, groups of 25, were too large for students to be fully willing to air their difficulties.

2. Pace of Class

In a normal class the instructors have a responsibility to teach all students and however sympathetic and interested they are in those with difficulties, the body of students has certain expectations which one must try to meet. So one is forced to help the stragglers to overcome their difficulties as quickly and effectively as possible - one has to help them to jump hurdles, whereas in the teachers' courses we were able to spend perhaps several sessions digging deeper and deeper to find out the fundamental nature of those hurdles and how they had been built up.

Even in the lab/discussion sections, time for clearing up difficulties was too limited, because too much time was spent on one set of difficulties meant that these sessions would get behind the lectures. So the next set of problems were not being dealt with as they arose.

3. The Value of Participation in these Courses

In spite of these limitations, our involvement in these courses was useful because it gave us a broad survey of a range of problems, some expected others still surprising. Only by continued observation of this kind at all levels can a catalogue be built up of the barriers that exist, on the one hand to provide the raw material for more intensive study and on the other to guide curriculum developers.

Had time permitted, we would have involved Hopi and perhaps Sioux and Navajo teachers and children in our experimental teaching because the different structure of their language may lead to a different view of the physical world and a different intuitive understanding of its workings. The Navajo language, for example, where different verbs are used according to the nature of the object acting or acted upon, must influence its users in a different way from that in which English users find their thoughts shaped by the English language.

Subject Matter for the Courses

The topic of energy was fixed for the undergraduate classes.

For the teachers classes the topics chosen were:

Size and Scale,
Heat,
Light and Color.

Any scientific matter was open to investigation. However, with the limited time and facilities available, it was necessary to lean on previous experience and choose topics where we knew that difficulties were likely to arise. It was our experience, and that of others, that in biology and the earth sciences many difficulties were those connected with mathematical and physical concepts.

Arrangements for the Courses

Allowing for the heavy pressures on teachers at the beginning and end of the semester, it appeared that one or two courses spread over not more than a twelve-week period would be satisfactory. The Size and Scale course occupied 8 weeks, Heat 6 weeks, and Light and Color 11 weeks. One meeting per week was the most that could be expected of teachers, and two hours was a suitable length of class, allowing sufficiently time for the teaching to proceed at a satisfactorily leisurely pace while not over-tiring teachers at the end of a day of teaching.

Within each class we allowed time for:

a) initial presentation and discussion of new subject matter and/or discussion of the previous week's work;

b) practical work either as a group or as individuals or in ones or twos, with the research team circulating, observing, questioning, helping;

c) final discussion of the work of the session and writing of brief notes on reactions to that work.

If individuals were to be observed and helped, ten to twelve was the maximum class size for a research team of three or four.

Staff meetings had to be timed to allow for discussions of the previous session's work and to allow time for the preparation of materials and equipment for the following session in the light of the needs and difficulties exposed earlier.

Reflective written notes were required of each participant following each session. To encourage the writers, and make it clear that their efforts were valued, it was decided that they should be collected and copied between sessions and handed back with written comments from each member of the research team at the subsequent session.

Collection of Data

Since we were interested not in whether our subjects got answers right or wrong but in the nature of their difficulties and the style of thinking that produced them, our data would be anecdotal rather than statistical. We would record their expressed difficulties as completely as possible. This was achieved by:

a) Working with individuals and groups, noting and discussing problems as they arose in dealing with the investigation under way.

b) Tape recording class discussions.

c) Getting students to write their immediate reactions briefly, after each class.

d) Collecting more lengthy, reflective reactions to each class in the period between classes.

e) Individual interviews with participants.

f) Reflective notes from participants about the whole course.

Complete transcriptions of class discussions and staff meetings, together with copies of participants written commentaries appear in Volumes

During the practical work periods, when participants and research team members were often scattered and working in small, often changing groups, tape recording was difficult and would, it was felt, have intruded into the informal relations between those involved. It seemed better to rely on personal observations and recollections of these occasions, rather than risk disturbing the atmosphere by attempting to obtain verbatim reports.

Analysis of Data

Had the project been funded for the three years requested instead of eighteen months, we would have attempted to work through this analysis:

a) Extraction of specific examples of difficulties from records.

b) Analyzing records for frequency of occurrence of difficulties.

c) Sorting difficulties into groups showing common characteristics, for example: Those brought about by teaching, confusing language, and confused spatial ideas.

d) Looking for connections between groups.

e) Hierarchical arrangement.

f) Comparison with McDermott and others. In each case there would be individual analysis by each team member followed by evaluation by the whole team to seek consensus.

f) Suggestions for curriculum improvement based on findings.

In the limited time available it was not possible to make the fullest possible use of the data which we had collected, nor were we able to follow the process of analysis which we had intended to use. Instead each member of the project team has produced one or more essays, reviewing the work from a different viewpoint.

"Introduction & Summary of Some Findings" by David Hawkins

This essay contains a discussion of critical barriers in the context of a general account - derived mainly from the thought and language of William James - of educationally significant science learning, learning which involves some essential conceptual reconstruction. In this context I have chosen, for illustration, three topics from our teaching: Size and Scale; Heat and Temperature; Light and Color.

"A Preliminary Taxonomy of Critical Barriers" by Abraham S. Flexer

In this essay Mr. Flexer describes the three principle categories among which we propose to partition the critical barriers encountered during the project's research. The first is a set of pervasive critical barriers that were encountered in so many different contexts that we suspect them to be independent of subject matter. Two other categories include less pervasive critical barriers: one category is discussed in this essay; the other in the essay "Problems With Mathematics" by Ron Colton on barriers related to mathematics.

"Problems With Mathematics" by Ronald Colton

"Problems With Mathematics" outlines the main mathematical difficulties encountered during the research, with special emphasis on problems with geometrical concepts. While difficulties with computation may impede the students' ability to solve problems or to follow the instructors numerical ability to solve problems or to follow the instructors numerical example, lack of a secure grasp of certain geometrical principles may prevent even the non-quantitative understanding of basic sicentific ideas.

"Ecological Contexts of Critical Barriers" by Abraham S. Flexer

This essay discusses the background against which critical barriers develop and must be dealt with, and propose the term ecological context to include the components of that background. Three large categories of ecological contexts, each with several subcategories, are described. Each category and subcategory is documented with examples from the project's research.

"Clues from the History of Science" by Abraham S. Flexer

In "Clues from the History of Science" Mr. Flexer argues that there are important historical and biographical reasons to expect parallels between early scientific theories and personal conceptions about the world that children bring with them to the classroom. Further, discrepencies between the personal conception and contemporary scientific views are may lead to critical barriers. Implications for education research, especially research on critical barriers, and for improving pedagogical practice are discussed.

"The Students' Views" by Maja Apelman

In this essay I have given a detailed description of eight class meetings dealing with the topic of Size and Scale. Since most of the literature describing students' difficulties in learning science is presented from teachers' or researchers' points of view, this essay attempts to show how a group of teacher/students perceived the content of one science class and how they thought and felt about the teaching approach. Many quotations from the class participant's notes as well as from the transcripts of classes and staff discussions attempt to convey the atmosphere of this seminar and thus to complete the more theoretical discussions of the research report.

"Notes and Reflections on Course: Man and Nature: Energy" by Maja Apelman

In this essay I briefly discuss the aim of a college course which was used for part of the research. Then I describe my personal experience as participant observer emphasizing problems which I encountered and which, I believe, will throw light on difficulties typical of many adults who come to a science class with little or no background in science.

"Man and Nature - Energy Courses" by Ronald Colton

This essay briefly lists a number of repeated difficulties which students encountered during the two "Energy" courses and comments on the problems encountered by members of the research group in using these courses as a medium for the type of research with which they were involved.

The Faculty Seminar

This was to play an important part in the project, in discussing our findings at various stages and suggesting further stages of inquiry. As it was, the shortened time-scale of the study prevented this from happening. We now recognize that we have just reached the point when the seminar group could have been useful. Our early meetings, before we moved very far with our investigations were necessarily based on generalizations, on our previous experience as individuals, and on the personal interests of the group's members, drawn from outside the project.

It is now clear that, if we were able to present the group with our assembled raw data, and discuss with them our reflection on it, we would have very rich material for discussion, each member would be able to contribute from his or her professional standpoint. They would then play the role that we had originally envisaged, helping to analyze and digest the data and formulating new procedures for delving deeper into the difficulties we had uncovered. Perhaps only now are we ready to make full use of their knowledge.

Mountain View Center
Research on Critical Barriers,
Fall 1981 - Spring 1982

Conceptual Barriers Encountered in Teaching Science to Adults:
An Outline of Theory
and a
Summary of Some Supporting Evidence

by David Hawkins

PART I

Introductory Statement

In the normal course of science teaching at elementary levels -- whether with children, adolescents, or adults -- many kinds of learning difficulties may be encountered. In our proposal, we singled out a special category of difficulties we wished to investigate, which we called critical barriers. Our working definition has two parts. First, critical barriers are conceptual obstacles which confine and inhibit scientific understanding. Second, they are critical, and so differ from other conceptual difficulties, because: a. they involve preconceptions, which the learner retrieves from past experiences, that are incompatible with scientific understanding; b. they are widespread among adults as well as children, among the academically able but scientifically naive as well as those less well educated; c. they involve not simply difficulty in acquiring scientific facts but in assimilating conceptual frames for ordering and retrieving important facts; d. they are not narrow in their application but, when once surmounted, provide keys to the comprehension of a range of phenomena. To surmount a critical barrier is not merely to overcome one obstacle but to open up new pathways to scientific understanding; e. Another hallmark of the class is that when a distinct breakthrough does occur, there is often strong affect, a true joy in discovery.

The difficulties thus singled out are difficulties resulting from an apparent mismatch between two conceptual modes: that which is communicated or presupposed in the normal instructional process, and that which is, in biographical fact, accesible for recall and use by a learner. Such research is important for the general light it may shed on questions of intellectual development and of learning. It is practically important if it can lead to significant improvement in the art of science teaching.

Our general category of critical barriers includes, as we see it, some of the most refractory problems of science education, problems which are least likely to be understood and solved without some major research investment.

This category of conceptual barriers appears very nicely to include a number of studies already in the literature (see Appendix A). In our proposal (see Appendix B) we have listed numerous other examples which have appeared repeatedly in our own work with elementary school-age children and, especially, with their teachers. We have studied these phenomena informally and formulated various conjectures about their nature and origin; above all we have tried to develop a style of teaching which is responsive to our students' need to deal with these barriers when recognized as such.

In contrast to most related studies we have seen our work has three distinguishing characteristics: it is *informal*, *naturalistic*, and *extensive*. It aims to explore a fairly wide territory, collecting specimens and describing them as carefully as possible, formulating hypotheses about them for later study where possible. We believe much more such work should be done across the whole range of science teaching. There is a classical paradigm for such research, though of course on a far grander and more impressive scale than anything we alone have tackled. It is the nineteenth century voyage of HMS *Challenger*, a worldwide search which yielded 4,000 new marine species and laid the basis, in the end, for forty monographic volumes, penned by such investigators as Thompson, Huxley, and Haeckel.

One further contrast is worth comment. The *natural world* we have wished to explore is the world of elementary science teaching, mainly informal in style, oriented toward observation, construction, and experimentation. This teaching commitment imposes obvious constraints on our style of investigation but also creates a new opportunity. As teacher, even as Socratic teacher, one is predominantly a participant; what can be learned from observed and observant teaching is in some contrast with studies which have the important but more restricted aim of "paying attention to what students don't know."*

Our purpose is always, at least in principle, to find out conjecturally, and more firmly where possible, what students do know, and then how this knowledge can be raised by them to the level of consciousness -- retrieved for their own use in further learning. Thus, for example, many scientifically naive students will have some understanding of balance, but will be unlikely to make use of this knowledge in coming to an understanding of the siphon or the barometer. Directed first-hand experience, such as play with air and with hydrostatic balance phenomena, can provide the missing link; or that link may be provided in other ways.

It should be said quickly that the evidence from our record, of our own teaching skill, when judged by the above criterion, shows it to be very uneven. We ourselves have seldom before been afforded the slow pace and deliberate aim to unearth and analyze student difficulties, and to do so in a kind of seminar atmosphere. In spite of the slow pace and self-conscious discussion with our students, we often went too fast and overlooked clues which the record reveals or at least suggests.

Indeed, the concrete situation of an investigative teacher may suggest another paradigm of research which has only been honored in special situations. It is not simply that of the collection-minded naturalist, but that of a reader of signs, a Sherlock Holmes or a Sigmund Freud, or perhaps a primitive tracker and hunter versed in the art of reading footprints.*

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* Milkent, Marlene M., "It's Time We Started Paying Attention to What Students Don't Know", Science Education, 61(3): 409-413 (1977).

In the present paper I have undertaken a kind of overview of some of the kinds of critical barrier phenomena we have observed, repeatedly and more or less predictably, among scientifically naive learners. But our work has raised numerous other questions than those we initially promised to focus on; the other essays in this report deal with the same year-and-a-half experiences, but from viewpoints which their titles make clear.

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* For a fascinating discussion of this paradigm see Carlo Ginzberg, "Morelli, Freud and Sherlock Holmes: Clues and Scientific Method", translated by Anna Davin in History Workshop, No. 9, Spring 1980.

PART II

1. Sagacity, Learning, and Taxonomy

Recent research on learning, problem solving, and cognitive development has brought us back, with renewed vigor and -- perhaps -- fresh insights, to some of the oldest problems of the theory of knowledge. For the purposes of our present research it will suffice to go back to the end of the nineteenth century, to William James' classic Psychology.* James' discussions of thinking and of reasoning are all pertinent to the present-day concerns of science and mathematics teaching, and more generally to the investigation of the cognitive procedures of children and adults.

James makes an initial distinction, reflected in many languages by contrasting verbs or nouns (kennen and wissen, connaitre and savoir, etc.), and uses a Jamesian terminology to mark this contrast: acquaintance with versus knowledge about. Acquaintance-with implies familiarity, recognition. As a mode of knowledge, it is not attributive, not propositional in character. We always in principle know something about the things we are acquainted with, but this implies a focusing of attention and effort of analysis which mere acquaintance does not require. We are acquainted with some persons and not others, as with some situations and places. James' emphasis on and uses of this distinction prepare the way, in his subsequent chapters, for a certain dichotomous tension which appears in different guises in different contexts: between perception and conception, between particulars and universals, between the peripheral and the central, between the concrete and the abstract, the intuitive and the analytical, the figural and the formal. When James turns, in vol. II. to the treatment of reasoning (our current jargon would usually, more narrowly, say problem-solving), he places an essential emphasis on the contrast -- related to that of acquaintance-with versus knowledge-about -- between sagacity and learning.

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* James, William, Psychology, 2 vols., Henry Holt & Co., New York, 1899. The most relevant chapters are VIII, IX, and XXII.

James' discussions of reasoning is cast partly in everyday psychological language and partly in terms of traditional Aristotelian logic. Reasoning is about something, it has a subject, a subject presented for thought. The process of thought is one of predication or attribution. The resources for thinking are always thus dual in character. The first resources are presentational (typically perceptual), the second are those somehow retrieved from the thinker's fund or store of knowledge. In making this obvious first move, James is aware of a question which is often overlooked. What is presented as subject for thought is typically a concrete thing or situation, a logical particular. But it is, and is perceived as, a particular of some kind, as exhibiting a logical universal. In order to retrieve or recall anything about the subject for thought, we must recognize in it, and single out, some universal character or trait, or implicitly grasp some similarity to things previously encountered which we can recall. This ability James, following John Locke, calls sagacity. If we think in terms of the mind's filing system, the perceived character of the subject is what directs us to some relevant file or files. In computer jargon it contains the address of such a store.

The outcome of thinking is learning: what is retrieved from the store and reliably fitted to the new situation is itself presumably what had once been learned and stored there. What is thus freshly learned is then also added to the store. Thinking is an interplay between sagacity and learning.

James' use of the term sagacity suggests, as it is intended to, that any concrete particular subject of thinking, attended to sagaciously, is attended to as an instance of some essential universal category: it is not only seen, but seen as an instance. This latter notion, of seeing-as, was recently emphasized by Wittgenstein*, who wished to avoid the paradoxes which can result when the perceptual and conceptual aspects of thought are too sharply separated. However veracious and undistorted our perception may be, it is always a partial and selective affair, thus made ready to fit some interpretation which context and habit make likely. And however abstract and formally defined our conceptual apparatus may be, it is always linked to some diversity of perceptual or figural material, to the imagery of recalled or imaginary perceptual experience, to intuition.

When the linkage between the perceptual and the conceptual (or the particular and the universal) is very strong and immediate, we approximate the kind of experience in which reasoning plays no part at all. Most of our daily performances, if observed and catalogued minute by minute, would appear to be of this kind, routine, habitual, unthinking. The stimulus and response are one seamless fabric. What presents itself for perception is immediately recognized and responded to in some more or less appropriate way, as a whole of meaning, familiar and unproblematic. "A rose is a rose is a rose..." There is no reasoning involved.

It seems reasonable to recognize, within experience, a sort of continuum, ranging from such virtual automatism at one extreme (in which things are taken at what we call face value and responded to unthinkingly) to a predominance, at the other extreme, of uncertainty, of awareness of novelty, of recognition of the problematic, and thus on occasion of the supervention of a new level of activity, of more or less systematic and analytical thought.

James' recognition of the sagacity-learning linkage invites us to a kind of taxonomic, and thoroughly Aristotelian, view of what Plato called coming-to-know, of the development of our ability to learn, to store what is learned, to retrieve what has previously been stored, to return what has been freshly added to the store and, as a not infrequent consequence, to work at reorganizing the storehouse itself.* In our present view, all of the important kinds of learning difficulties we (as reflective teachers and researchers) have encountered can be described within this scheme.

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* Ludwig Wittgenstein, Philosophical Investigations, New York, 1953.

James' emphasis on the notion of sagacity, therefore, raises a question which is absolutely central to all of our concerns about teaching and learning. How is it, and under what circumstances is it optimally possible that we are able to gain access to previously stored information which will, in new and problematic situations, help us on our way to problem solutions or, more generally, to fresh understanding? And how is it that, in the wake of failures, we are able, with help, to reorganize some parts of the store, to add to them fresh experience, and so to find successes?

In the Aristotelian scheme, a mind's fund of knowledge is organized _per genus et differentiam_, as a taxonomy, as a filing system in which each genus is subdivided by differentiating characteristics into two (or sometimes more) sub-genera. The defining characteristics of each taxon, each genus, are chosen, but only more or less adequately, to be those which are _essential_; that is, to be just those characteristics which are most reliably associated with many others. Thus in Aristotle, the category _man_ is a subdivision under _animal_ distinguished by _rational_. The genus _biped_ and the differentia _featherless_ would equally well distinguish us from the other animals, but would not provide or sustain a rich or logically coherent taxonomy. Very little important information about humans would be related merely to our bipedal status, and even less to our lack of feathers. Animality, on the other hand (we moderns would qualify and say mammal, or even primate, animality), supplemented by some label which recognizes our special capacities for communication and learning, would provide a far more useful (because more coherent) organization of what we know about man's place in nature. Such central attributions are "of the essence" in Aristotle's scheme.

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* cf. Hawkins, D., "Taxonomy and Information", _Boston Studies in the Philosophy of Science_, III, 1966.

James' distinction of sagacity and learning is related to a classic discussion of Aristotle (e.g., Metaphysics, Book I, Ch. 2). In discussing the nature of wisdom, he defines a sort of continuum of things knowable, lying between two extremes. At one extreme are those things which are "most knowable by us" and at the other are those which are "most knowable in themselves." The latter are exemplified by first principles, laws, universal truths. They are first in the order of importance, but last in the order of learning. Because of their abstractness and universality, they relate to and are involved in defining the most generic features of that which we experience. What is most knowable by us, on the other hand, coming first in our experience, is the world of the concrete particular phenomena and the wide diversity of quite specific categories into which these fall.

2. Common Sense to Science - Category Shifts

One of James' central arguments concerning the categorization of experience is a kind of modern relativism which Aristotle would not share or find relevant to his concerns. James recognizes the possibility of many different taxonomies, insisting that what we regard as the essential characteristics of things is wholly relative to the dominant purposes for which we use or take account of them. By way of comment on Jamesian relativism versus Aristotelian metaphysics, it is useful (and sufficient for our present study) to take note of a characteristic contrast, in implied purposes, between the commonsense organization of knowledge, its category structure, and that of the sciences.

Commonsense categories tend to be defined (implicitly) by relatively accessible characteristics and, where these are not reliable indicators of other consequential traits, we tend to define by clusters of such characteristics, figurally rather than formally*; by some kinds of perceived pattern-similarity, "family resemblance". To quote Eve, in Mark Twain's story of the newly created Garden, "It just looks like a camel." We are told that Massai herdsmen can recognize and sort each others' cattle, in large herds, without resort to branding. Apparently personal ownership becomes reliably associated with pattern-differences across a large diversity of subtle variations. The reliable, rapid reading of a printed text for understanding is perhaps an achievement of the same order.

Another characteristic of commonsense categories is the generally loose logical organization in which they are related to each other. Thus, larger classes are usually recognized and identified only by practically accessible characteristics or clusters. So we can often recognize and identify some individual species of plants, but their genera and families are totally unrecognized unless they happen to be grouped and distinguished by some simple or obvious traits. But these in turn are often at variance with the scientific groupings, analogous rather than homologous. Thus a whale is a fish with a horizontal tail, as it says in Moby Dick. Such groupings are obviously relative to characteristic human interests and purposes. So also, of course, is the alternative classification of whales as cetacean mammals rather than as any kind of fish. It is difficult, however, to regard the existence of such alternative ways of classifying as a demonstration of relativism when so little of closely observed whale behavior is fish-like, and so much else fits the description of a mammal evelved to be ocean-going. In Aristotle's language, this second description "divides nature at the joints", which the first to a degree does not. Thus we can admit that classification is relative to the purposes for which it is organized, but we can observe, all the same, that some classifications can serve a far wider variety of purposes than others including, most importantly, the further pursuit of knowledge.

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* See Jean Bamberger and Donald Schön, "The Figural ⟷ Formal Transaction", DSRE Working Paper WP-1, MIT Room 20C-124, June 1978.

The history of science may thus be regarded as a history in which the filing systems of commonsense knowledge are (a) deliberately expanded in content, (b) reorganized (sometimes radically) to accomodate this expanding content with minimal redundancy, and in which (c) each of these commitments is deliberately chosen as a guide to the pursuit of the other.

The whale is one example of this kind of category shift. Another example of this process, characteristic but very simple, arises in the elementary understanding and terminology of plant anatomy. Having previously recognized the distinction between simple and compound leaves in such obvious cases as the locust, and having observed the universality of the bud at the axil of the true leaf, one is then obliged to say that what common sense would immediately recognize in shape and size as leaves (eg., in the Kentucky Coffee Plant) are really only leaflets, small parts of the true leaves, which common sense would in turn call branches. In this reconstruction one sees an absolutely characteristic scientific motive, which common sense does not often share or need to share -- a motive of loyalty to universals (in this case of plant anatomy and development), many of which are verifiable only by far closer examination than our normal prescientific interests would sustain.

A more complex case is one which surrounds the concept of metal*, as that has been reconstructed throughout the history of chemistry and physics. The common sense, ancient conception of metal can be more or less identified with some cluster of relatively obvious characteristics (shiny color, heat conductivity, etc.), a definition by family resemblance. The twentieth century scientific conception, on the other hand, is rooted in that of an atomic crystal lattice structure in which the outer electrons are very easily detachable within the lattice, forming a kind of "electron gas." This conception is defined in terms of characteristics which are radically inaccessible to common macroscopic experience, but which, when defined, allow an extensive and precise elucidating of properties, linking these into the generalities of quantum physics.

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* cf. John Dewey, Logic, The Theory of Inquiry, New York, 1939, Ch.IV, passim.

If we now return to the Jamesian style of discussion we may construct a very skeletal account of levels or phases of thought (reasoning, inquiry) involved in the use, retrieval, and reorganization which is implied by this sort of transition from common sense to scientific categorization. To carry out this discussion, we extend a scheme which James only mentions in passing, that of the traditional patterns of the syllogism. The extension lies in the use we make of these patterns, as follows:

1. In the case of automatic of unmediated recognition we write:

$$S \longrightarrow P \text{ (Sagacity)}$$

meaning simply that some presented situation S is seen as an instance of P and responded to directly, with no further thought.

2. In the next level this operation of sagacity occurs, S is seen as of some kind, M. But M is now not of itself sufficient to close inquiry; it serves instead as an address to a whole file of things labeled M; and from that file is drawn a generalization: Things called M may be relied upon to have the property P. With this discovery the inquiry is closed:

$$\begin{array}{l} S \longrightarrow M \text{ (sagacity)} \\ M \longrightarrow P \text{ (learning)} \\ \hline S \longrightarrow P \text{ (Closure)} \end{array}$$

This is, of course, the traditional first figure of the Aristotelian syllogism. In the logic text, this figure (called subsumption) looks rather trivial: "Socrates is a man, all men are mortal, therefore Socrates is mortal." If however, we are simply using this pattern to *describe* a common pattern of reasoning, it fits well enough. The new situation S is *seen as* a case of M, but the M in question may not be quite the same, in numerous respects, as those past cases of M which have supported the generalization that all M's are P. In this case we might write

$$\begin{array}{l} S \longrightarrow M' \\ M \longrightarrow P \\ \hline S \longrightarrow P? \end{array}$$

and thus indicate that an element of judgment and hazard (as to the importance of the difference between M' and M) may be involved. Whether this judgment is finally confirmed or shown wrong, the content of the file M will have been altered by the inclusion of a new instance, and also therefore by the strengthening, rejection or redefinition of some generalization based upon the collection of instances of M.

In fact it is clear that the pattern

$$\begin{array}{l} S \longrightarrow M' \\ M \longrightarrow P \\ \hline S \longrightarrow P\ ? \end{array}$$

leads very simply to a consideration of *analogy*. M' being no longer taken immediately as identical with M, we examine to see if the identifiable *differences* between the present M' and past instances of M are or are not relevant - whether the *grounds of analogy* are weak or strong.

In the case of biological taxonomies, these alternatives are often represented by a contrast between analogy and homology, between accidental similarity and similarity due to common origin. So the whale and fish have some analogous characteristics, but the pentadactyl limb is a homologous link between whale and land-mammal. Biological influences based on mere analogy may be limited and superficial; those based on homology are deep-going. The Tasmanian wolf is not a wolf but a marsupial, analogous to the wolf in appearance and habit, but only far more remotely homologous. When common sense habitually and dogmatically classifies by superficial appearance, we may call this habit the Tasmanian Wolf syndrome.

3. At a third level, having seen that S is M'; and having discovered that the file M is not a useful guide to thought, or to further inquiry, a next possible step is more careful examination of S itself, in which S is seen finally, to be of a kind genuinely different from M -- say, N. N is now the index to quite another file in the store of knowledge, and the process starts over again.

A simple example is the transition, discussed below (p. 38) in the investigation of the hot-cold contrariety, from this kind of polarity to a conception of heat as a physical substance of some kind; under this newly-tried category, one can now conjecture that there will be a quantity of heat in any material thing which could approach zero and thus, also, imply an absolute zero of temperature.

4. The most characteristic use of analogy in thinking arises when there appears to be no general file category related to our most sagacious perceptions of the situation S. S now presents itself as uniquely novel. Lacking any general catagory to fit it in, we can look for other particulars we can recall from memory; these may in turn create new direction for

file-searching. Thus a mathematical problem, not soluble by any known algorithm, may remind one (on casting about) of *another* problem, superficially dissimilar, which *has* yielded to some particular method of solution. The first problem is now tentatively *seen as* an instance of some quite different trait, no longer as an S that is M or M' but as a T which is thereafter found to be an instance of R; so the file-search process is begun all over again.

As a result of this kind of reconstruciton, the acquisition of scientific knowledge creates a special difficulty -- that scientific concepts often form an interconnected network. Any *one* such concept is to be understood as a node in a network involving *other* scientific concepts, tightly interconnected. Commonsense concepts, by contrast, are often loosely connected, since they are defined by some readily observable traits, or by family resemblances among clusters of such traits. Such concepts are thus relatively less dependent on their logical interrelations to each other, more readily established one by one, "most knowable by us."

In examples such as the solid-state physics theory of metals, one can observe the very great organizing power of theory (electromagnetism, the quantum atom, crystallography, for example), providing as it does a relatively small collection of conceptual tools which provide a description of nature over very wide ranges. At the same time, however, one can find at least a partial explanation for many of those apparently rudimentary learning difficulties with which our research is concerned. Scientific concepts, in contrast with those of common sense, however illuminating when well-understood ("most knowable in themselves"), often form a network which is **strongly** interconnected. *To understand any one concept, a node in the network logically connected to other nodes, it is necessary to understand many others as well.*

This logical tightness in the network of scientific ideas, their mutual interdependence, suggests immediately a paradox: they cannot be learned: not in isolation from each other, not all at once, hence not at all. Such a paradoxical conclusion only states, in extreme form, the origin of many of the student difficulties.

cf II, p 72

3. Model Building and Model Testing for Critical Barriers

As the forgoing discussion implies, we propose to describe, and in a sense explain, some characteristic difficulties in the learning and teaching of science, and to do so in a language which discusses phases and transitions of experience, building models which represent what we have called critical barrier phenomena. Such models purport to describe interior processes which will be reflected in learner's observable behavior.

One question about such thought-models is whether they can, as hypotheses, be adequately tested by empirical data, or whether they will remain only ad hoc, speculative accounts.* We shall give no dogmatic answers to this question. Our efforts at confirmation of some models has been informal, exploratory. But we think we have turned up some interesting clusters of phenomena, some of which we have been able to predict from others on the basis of fairly simple explanatory models (see below, Part III).

A second question about such thought-models concerns their usefulness as guides toward the improvement of teaching. In the twentieth century history of psychology, such accounts of thinking and learning as we here offer have often been disparaged as "introspective" rather than behavioral. As this applies to our work, we reject the label. It would be better to say that the models we develop are models which impute to our students the same capacities which are implied in our own investigative performance. When, for example, we try to learn about learning we -- as researchers -- exhibit the strengths and limitations of our own sagacity and our own learning; the conceptual apparatus and language we make use of is of the same genre as that of the persons we study. Their thinking may be more scientifically naive than ours in some ways, but it does not differ in kind. We impute ways of thinking to them which we ourselves can try to practice and report on, and which can give guidance to our teaching.

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* cf. Robert Davis, "The Postulation of ... Frames." The Journal of Children's Mathematical Behavior, Vol.3, no.1, Autumn 1981, pp.167-201, esp. pp.167-170.

If we find significant results in this mode then also, we believe, these results are directly useful for other teachers who have understood them. If many students have difficulties in the same ways and around the same subject matter, thoughtful teachers will attempt just the same kind of model-building which such research as ours can pursue further, in a more careful way. This will be to the further benefit of teachers' diagnostic and planning abilities.

The general outline given in Parts 1 and 2 above -- in the name of Jamesian psychology -- is not logically tight enough or detailed enough to be called a testable theory. It is a necessary sort of plausible framework of conditioning assumptions. It directs us to look for the typical source of critical barriers in the reconstruction of mental filing systems, those which scientific understanding requires; and in the kinds of category-shifts which this leads to, which can then be consolidated as a basis for greater scientific understanding. In the following sections I shall set forth examples, both from previous teaching experience and from current (and more carefully documented) research. In some cases we merely describe certain characteristic learning-difficulties.

In a few others we go further in developing and giving evidence to support some of the kinds of unifying hypotheses which can help explain a range or cluster of such difficulties.

4. Teaching

The above account of cognitive procedures, of thinking suggests a final methodological comment. Our conditioning assumptions imply that the most persistent and basic difficulties in the acquisition of scientific knowledge are difficulties which involve not so much a sheer lack of information as a trouble in making category shifts, shifts which involve a reconstruction of ways in which experience is codified and filed away, and then later retrieved. If this is a correct assumption, then an essential phase of the teacher's art must be conceived of supporting and seeking to guide the learner's own reconstructive commitments and efforts. Where such shifts are needed, a teaching style which simply transmits more scientific information, more knowledge, into the learner's already established but inappropriate taxonomic scheme, may prove radically inefficient, even pedagogenic. If the learner's taxonomy is seriously at variance with that which the teacher's instructional efforts presuppose, only verbal and conceptual conflation and confusion can result; only troubles of a pedagogenic order will accrue.

cf M.A., "The Students' Views," p 14

It is not yet our major purpose in this research to define ways of teaching which will prove appropriate to motivate and support such reconstructive achievements. As will be clear from the transcripts of our courses for elementary-school teachers, however, we have in fact taught in a style which allows a great deal of time for exploratory play and observation, and which, for our part, emphasizes the questions and puzzles which touch some of the conceptual difficulties involved. We cf III, p.7

have tried so far as possible to create an atmosphere in which confessions of uncertainty and confusion or assertions of belief seemingly contrary to the scientific were as much of interest as were bursts of new scientific insight.

One of the consequences of this commitment has been a radical deceleration of the rate of coverage of subject matter. In our teacher seminars the three topics of size and scale, heat and temperature, light and color, occupied us for some fifty contact hours, and perhaps covered -- we hope uncovered -- the equivalent of three elementary textbook chapters. Our teacher-students, socially mature but scientifically innocent and in some measure "turned off" by science, understood the reason for this pace, which was to create situations which would elecit reflection and discussion about our individual efforts to comprehend our chosen subject matter. On the whole, we believe, this slow pace helped to create and maintain the needed atmosphere (see esp. teacher commentary, vol.IV,pp.239-79). On the other hand the teachers were sometimes upset by our inability, or unwillingness, to go on to more advanced explanations than our lab work and discussions could make meaningful.

In the two undergraduate courses, both titled "Man and Energy", this radical deceleration was not possible, and for a variety of reasons we were unable to carry out the interviews with students which we had originally planned. Two of us participated in teaching a lab-discussion section and occasionally lectured, one for one semester and the other for two. This course was taught in a far-from-standard style, with much effort devoted to the cultivation of simple physical intuition and to the arts of order-of-magnitude thinking. Our major harvest from its program is a substantial and detailed record of undergraduate difficulties with extremely elementary geometrical relationships and with the uses of number in the discussion of physical quantities (for example, translation of units, areas, and volumes, multiplicative processes). Some of this material overlaps with things learned from the teachers' courses. In our original plan for a three-year research project, we aimed to examine the same range of conceptual topics across the age-span of children, undergraduates and teachers. In the year-and-a-half allowed us to demonstrate the feasibility of our approach, this ambition had to be radically restricted.

PART III. THE FINDINGS

1. Introduction

The record of this work, written or transcribed, some 700 pages in length, is far richer than our current and time-limited use of it can exploit; we all hope to return to it in our later work. In the present essay I seek to illustrate and, I hope, justify a kind of naturalistic detective work, by which potential barrier phenomena first come to teachers' (as to our own) attention, and then, to stimulate conjecture on the nature and origin of the apparent conceptual difficulties. The sources of this conjecture may be varied and numerous: personal recollections of one's own or previous students' troubles, intellectual struggles recorded in the history of science, analogies from everyday experience, and possibly others.

At the beginning of this paper I gave an operational definition of Critical Barriers, in terms of student's resistance to ordinary expository and explanatory modes of teaching, in terms of their own often suggestive acknowledgements of confusion, and in terms of the evidence of strong affect when such difficulties are finally overcome. To recognize a student's difficulty as potentially of this kind has involved us in learning about various other sorts of learning difficulties than those we have promised to investigate. Before we undertook the present investigation, we had rather overlooked the obvious need to filter out those difficulties which are not intrinsically conceptual. They may arise simply from distracting preoccupations in the life of a student, from poorly planned teaching not matched to the student's prior learning, from overly ambitious teaching, including especially the always-present didactic tendency to push ahead too rapidly before previous steps taken by students, correctly but without adequate consolidation, have been properly enjoyed and tested. Some of these difficulties are indeed pedagogenic -- the precise analogue of iatrogenic illness. In treating one illness of a patient, the doctor's ministrations unwittingly contribute to another, sometimes more serious. So a teacher, concerned to contribute to a student's understanding, may use his benovolent authority to create new confusions, importing explanatory ideas for which the student is even less prepared than he was for those which first occasioned trouble.

We have concluded, however, that certain kinds of conceptual difficulties which students experience are indeed intrinsic to the growth of scientific understanding and must be understood as such, rather than as some accidental result of informational overload, or of inadequate teaching. In the sections which follow we have set forth a sketch of several case histories, derived mainly from our two most recent teaching episodes. The first of these case histories is abstracted from our most recent course for elementary school teachers, entitled Light and Color, which extended over some two-hour, 25 hours, a once-a-week, late afternoon course.

2. Lux et Lumen*

We selected the Light and Color topic because we knew from previous small samples of its content that it proves rich in conceptual problems even when taught at a very elementary level. The sequence of topics was organized in accordance with a maxim which rather reverses the usual pedagogic order: Begin with the investigation of phenomena which are complex and rich in possibilities even if, as is typically the case, these phenomena would come late in the normal textbook order. For example: when introducing the phenomena of balance, with the final aim of understanding the additivity (the "law") of moments, do not begin with the case of equal weights in the equal-arm balance. Begin, rather, with situations which invite the placing of many weights which can be distributed on some three-dimensional support, yet will balance at some one point which can finally be located (for example, a broken-off tree branch). The obviously simple and symmetric case of equal weights at equal distances is initially uninteresting; it becomes significant only when early exploratory curiosity -- which feeds on diversity and complexity -- has been replaced by a more analytical interest.

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* See below, Section 3, p. 33-34.

Following this maxim, we introduced our teacher-students to, and invited them to play with, a wide range of additive color-mixture phenomena. The apparatus, typically consisting of three small projectors with a variety of colored theatrical gels available, was easy to use and inviting to play with. It happened that none of these adult students had ever been acquainted with the results of superimposing colored lights from different sources on one screen. They were amazed, quite uniformly, at the fact that this sort of (additive) color mixing gave results in radical contrast to those obtained with the much more familiar mixing of pigments. Already we were putting into their hands the basis of a contrast which even we, in planning the course, had not fully appreciated. This is the contrast between pigmented surfaces, seen by the way they selectively absorb and scatter ambient daylight or roomlight, and white surfaces, which scatter light emitted from one or more sources through colored filters, through slides, etc. In the first case, by far the most common, we all conceive of the color as a property of the physical surface. In the second case we think of colored light, and the way it makes things look or appear. From a third point of view perceived color depends also upon the nature of human perception, which maps all spectral distributions into the color triangle or, alternatively, into a threefold mixture of complementary pairs.

The record of our discussions around color is rife with questions unanswered, as it should be in view of the intrinsic complexity of the subject matter. (See Raw Data Volume IV, pp. 4-42 and Volume V, pp. 1-35) One fact is suggested very clearly by these early discussions. For most of us most of the time, color is understood primarily as a dispositional property of opaque physical things. Gold is yellow, leaves are green, violets are blue. These color properties are thought of as inherent dispositions and not as relative to conditions of illumination or perception. The conditions of illumination may affect how the colors of things appear, but not how they really are. Our adult students are aware that this is shaky ground, philosophically. It is practically firm, however, and they seem to have no reliable alternative scheme to put in place of it, even with projected colors.

One conspicuous puzzle concerned colored shadows. With two projector beams (say red and blue) a stick's shadow provided a variety of shadow play and proved endlessly intriguing and puzzling. With three projectors and the stick's shadow cast from each and from each pair (overlapping shadows), the seven colors visible can at first perplex us all. But with only two projectors, the red shadow cast by the blue light and the blue shadow cast by the red light, seemed a genuine puzzle, one which resolved itself only after much experimentation and discussion, and then with great excitement.

Only much later did we see this episode as providing evidence to support a general barrier-hypothesis developed from other sources.

This hypothesis had been formulated some years ago, and was given as an example in our proposal for the present research. When our former students (post-graduate students, elementary teachers, other miscellaneous adults, and elementary children) were asked to represent the geometry of their own oblique mirror vision with simple sketches, a majority represented the mirror image as located on or only slightly *in* the mirror. Self-viewing was similarly represented, both by incorrectly predicted camera focal distance and necessary mirror-size ("My ears are not *that* close together!").

In our unreported previous work, we had also frequently found ourselves at a pedagogical impasse. This had to do with different phenomena, the real images produced by pinholes or convex lenses. We have often provided students with a camera obscura: a pinhole- or lens-mediated camera big enough to get inside of, made from a refrigerator carton or closet with a hole in the door or wall, facing some brightly illuminated scene. When dark-adaptation makes the image visible, there is often great excitement, followed by a puzzle: but why is it upside down (and some notice, left-for-right)? (See Volume IV, pp.55-59 and Volume V, pp.61-80) Our own surprise, optically sophisticated, perhaps, but pedagogically naive, was that the standard ray-diagram, drawn on the blackboard, proved for the most part totally unenlightening, even confusing, in what we had thought was its obvious and transparent explanatory power. Our explanatory incompetence could be understood, we finally conjectured, if our students' perception of the image --trees and buildings, or friends jumping (upside-down!) and waving arms-- was being taken to be that of a *picture*, not significantly differentiated from the category of paintings or photographs or movies. If a gallery hung a Picasso madona-and-child upside down, the why-question would not be answered by a ray-diagram!

At first sight this suggestion seems remarkably implausible. Present-day adults and children see a plethora of images which are not cast in pigment, or which, if so, originated as emanated images cast in phosphors or in silver crystals. Yet from such evidence as we have accrued, it would appear that pictures are a single commonsense file category, not adequately subclassified.

What is crucial, apparently, is our students' failure to differentiate, in any essential way, between the color-pattern-cast-in-pigments, seen by scattered white light, and the emanated optical pattern, cast in rays of colored light and scattered from a uniform, neutral white screen. The latter kind of picture, of which our camera obscura images were composed, required for its intelligibility a new conceptual component which even in the modern world our common experience has not somehow provided: the physical-geometrical component called the light-ray, the essential contribution of geometrical optics.

Our next major step in the course, suggested in part by the complexities of color (in many ways a topic for separate discussion) was a retreat to the much simpler subject of light and shadow. Here, finally, we seemed to encounter a commonsense conceptual pattern which generalizes and unifies all the results suggested above, and embeds them in a kind of implicit theory of visual perception. In this conceptual pattern there are two quite distinct conceptions of light, not clearly related to each other. In one of these conceptions light is "the essential condition of vision, the opposite of darkness" (Webster, 1927). It is simply the (static) condition of vision. In some ways of thinking, darkness

cf
IV,176

is a sort of space-filling medium which makes it impossible for the eye to *reach out* to an object of vision, a medium dispelled in daylight or lamplight. Light in this sense is, so to say, the condition in which space becomes literally trans-parent. Thus, when the phrase *white light* is heard, one reaction is that "it sounds as though space was full of milk" (in an unrecorded episode, this was a strong consensus).

cf IV, p219

One of our associates, Ann Drucker, reports the following conversation between three fifth graders:

"If all the colors are in white light, then the sun must be white.!

"If the sun's light is white, then why isn't this room white?"

"Well, then, the sun must be clear."

"But how do we get colors then, because colors are in white light?"

"We *know* that because we saw red, yellow, and blue projected on the wall and they were white!"

"Well, then white light must be clear."

"Could white light and clear light be the same thing?"

"Then white light isn't *really* white light."

"The sun is yellow when you look at it though."

"The light bulb is white when you look at *it* though, and clear when it hits the air."

"Does white light change colors? It changes when it hits the air. Does the air act like a prism and change the light so that it doesn't look white? That's it, you guys!"

A conversation between Ted, Crissy, and
Ann November 24, 1981.

We could quote almost the same discussion among members of our teachers' seminar, but by chance have no tape.

The other contrasting and apparently subordinate conception of light is that of the light *of* or *from* the sun or any lamp, il-lumination.

It exists in rays or beams and can take on or project the color of a colored glass or theatrical gel. In the first category, the eye, the vision, "reaches out" to -- attains -- its object which it sees to be in front of it and more or less distant. In the second category, the ray or beam of light can enter the eye but when it does (and is very bright) it is seen *as* a ray or beam, as light, not as a distant object of visual perception.

It was, of course, this second conception of light which we pursued in connection with the topic of light and shadow. One of our initial steps was a simple technique of predicting, by a stretched string, where the shadow of an object would fall when the light was turned on. We had assumed, naively, that a string stretched from the light source to the screen, and moved around just tangent to the object whose shadow was to be cast, would represent the light-rays outlining the shadow-to-be. After some discussion, our teachers managed to instruct us: we were casually presupposing the central notion of geometrical optics, the rather high order physical-geometrical abstraction of the *light ray*, representing something which *traveled*, or the straight-line path *of* something that traveled, out in all directions from the lamp or sun. They simply were not ready for that; not for the idea that light *travels*, and not for its representation by rays or conical bundles of rays. Shadows are conceptualized instead, it would seem, as transient surface patterns of darkness (but not as three-dimensional, not as projections). Objects *have* or *cast* shadows, but the geometry is often vague.

Having been thus enlightened by our students, we then devoted substantial time to this geometry, which in turn took us back to the pin-hole image. The culminating experiment involved a thin plywood

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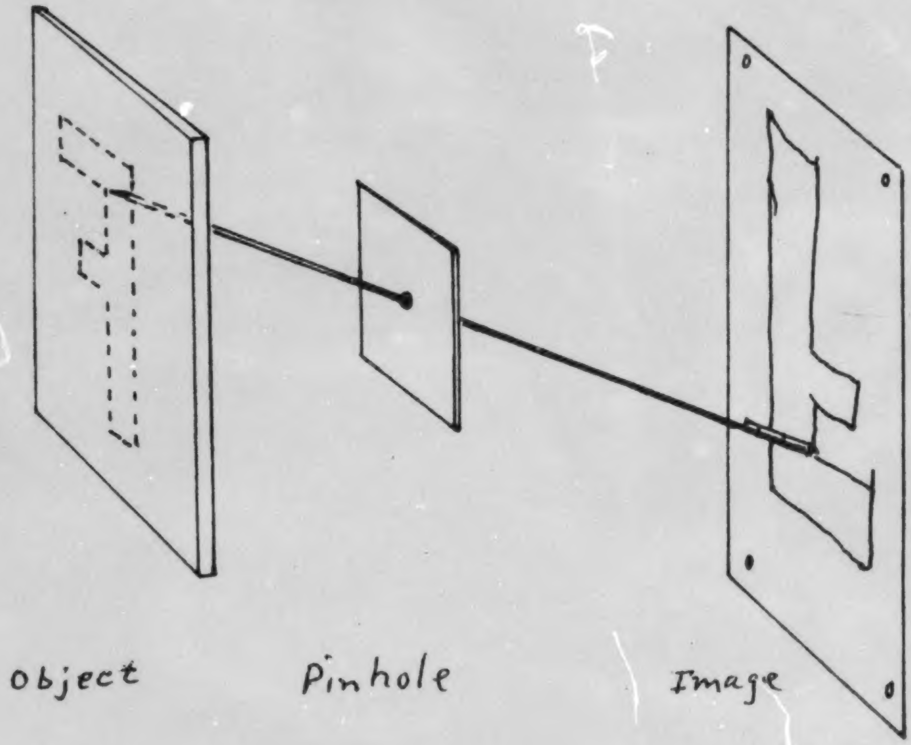

Object
Pinhole
Image

sheet with small holes drilled in it, in the pattern of a capital F, which was mounted three or four feet from a blackboard and parallel to it. Between this sheet and the blackboard was mounted a small plate with a single hole in it. A dowel could be pushed, or a penlight could be shone, through each hole in the plywood sheet in turn and, at the same time, through the hole in the intermediate plate, so that the stick touched, or a corresponding spot of light fell on, the blackboard at a point which could be marked. The result was not only satisfying, but exciting: "So that's how it works!" This reaction was shared by a majority -- all, I think -- though we kept no count. Not only did the secret of the pinhole image resolve itself, but also and incidentally, that of the shadows and even, for one or two who pursued it, of umbra and penumbra. The ray geometry was beginning to earn its keep. For example, the inverted chalk "image" of the letter F could now be either larger or smaller than the original, depending on the position of the intermediate plate - a possible path of entry into the study of magnifiers (Volume V, no.6, pp.81-93). Also IV, 185

Two sequels to this rather major development are appropriate to mention here. One was an extensive discussion, student-initiated, exploring the idea that the light in our room (light in the first of the senses defined above, the space-filling means of vision) could be conceived as an enormous congeries of light-rays (entities dashing about called photons, perhaps); these were constantly emitted from lamps or, as sunlight, entered the windows, were absorbed, or scattered about in all directions, sometimes selectively as to color; some entered our eyes to make retinal images which, though themselves unseeable, gave us means for seeing what we did see, those objects out in front of our eyes. This last was a mystery we acknowledged but did not explore, V,55-8 the epistemology of visual perception.

The second sequel is mentioned here only as a reminder that newly-developed conceptual tools can be fragile. When we finally took up the matter of mirror-vision, the light-ray geometry seemed to find no easy application to it, there was no transfer. We had indeed to live through all the difficulties mentioned above from work with others in previous years. In retrospect, a better pedagogy would have been to study windows first. "How can that big mountain get through this little window?" Only then perhaps would the reality of mirror-land have become acceptable.

V, 109 -112

I may perhaps mention that persons far more sophisticated scientifically than our teacher-students can still display the evidence of this deep-seated barrier. An old question (perhaps invented first for some PhD candidate in geometry or physics) asks: Why is it that when I look in the mirror I see my left exchanged for my right, but not my head for my feet? The story is told, by one of his students, that this question was once put to Niels Bohr. Bohr covered his eyes for quite a time, and then responded: "It is because I do not often stand on my head." The mathematics of the mirror-transformation, the reflecting window, is simple, yet - by the above kinds of evidence which we can easily accrue - deeply counter-intuitive.

3. Summary

The above informal case history is presented as an example of the kind of investigation, or detective work, which can lead to the construction and at least partial testing of models of conceptual difficulties common among present-day students of elementary science. Such models may then suggest new strategies of improved instruction. I give here an outline of our own detective-style model building, in the order in which, over several years, it has actually developed.

1. Phenomenon: Systematic misrepresentation of mirror-vision, suggesting that objects seen in mirrors are conceived as located at or near the mirror surface. First evidence: from students' graphic representation of oblique mirror vision, confirmed by data on size-distance judgments, predicted camera focal distance, etc. Hypothesis: Mirror images conceived as something like pictures. An alternative hypothesis is that in the dimension normal to the mirror surface, mirrorland is conceived as radically foreshortened, possibly "because it isn't real" (See Vol. IV., pp. 86-109 or Vol. V., pp. 108-115).

2. Phenomenon: Frequent great surprise at the fact that real images in our designs (created by pinhole or small long-focus lens) are upside down; and if this surprise is expressed, it is typically not relieved by an explanation which makes use of the standard ray-diagram. Hypothesis: the real image here, as with the mirror-image, is conceived as a picture on the wall or screen, one not essentially different from one cast in pigments; not as an emananistic pattern of light rays intercepted by a screen (See Vol.IV., pp. 61-66 or Vol.V., pp. 61-93).

3. Phenomenon: Colors of dual shadows from dual beams of contrasting colored light are not initially understood, or acceptibly explained by teacher. Hypothesis: Even after playing with two (or three) projectors and many colors of gels, students see the color-patterns on the wall as patterns of its surface, pigment-like or picture-like, rather than as emanations made visible by a white wall (See Vol.IV, pp.15-23 or Vol.V, pp.3-14).

4. Phenomenon: Everyday shadows are also not seen as occasioned by interrupted rectilinear propagation of light rays. Comment: Commonsense

may have such geometrical notions available, but somehow not readily. Socratic teaching needed! (See Vol.IV, pp.44-53 or Vol.V., pp. 36-60)

5. Phenomenon: Strong positive interest shown in physical, enactive ray-tracing which literally models the pinhole image (use of dowels, narrow light beams). Comment: Success! (See Vol.IV, pp. 44-66 and Vol.V., pp. 61-93)

6. Phenomenon: Reflective discussion of a quite new idea: that the "light" which pervades visual space and renders it penetrable to vision -- originally conceived as a kind of passive medium opposed to darkness -- is to be replaced, "scientifically", by the opposite and previously subordinate notion of evanescent discrete light rays constantly emitted from luminous sources and transmitted through space or transparent matter, reflected or scattered from surfaces (selectively as to color), and sooner or later absorbed. The rays, which are conceived, to start with, only as geometrical abstractions, thus acquire a quasi-material character. One must think of these rays, in daylight, as filling space densely, so that from any bit of source or scattering surface to any bit (for example from the sun to the surface to the pupil of an eye) there is always a bundle of rays. Somehow these rays are also endowed with color, a notion which again seems to violate common sense, which treats color as a disposition of material surfaces. Comment: this discussion seemed to me to indicate the emergence, and even the partial domestication, of a quite major category shift. Our students had learned almost nothing of physical optics, not even much of what one can do with purely geometrical ideas. Yet even to the learned, the photon is not much more than a ray -- a sort of detached Fourier coefficient associated with a frequency and a proportionate bit of energy. What our students had begun to learn

was a wholly new way of conceiving a range of almost-everyday phenomena. This new way of thinking has its roots in common sense but is given new power when associated with the geometrical abstraction of the ray, a directed line segment, and the implicit geometry which that abstraction brings with it (See Vol.V., pp.62-80).

The title we chose for the presentation of our optical case history - Lux et lumen - is taken from Gerald Holton's Thematic Origins of Scientific Thought (Harvard University Press, 1973). In Chapter IV, which can also be found in Daedalus (Fall 1970), Holton uses the Latin contrast of lux and lumen. Lux is what Webster calls "the condition - or medium - of vision", the opposite of darkness. The diffuse daylight before sunrise, which casts no shadows, is, so to say thought of as space itself, made penetrable to vision. The direct object of vision is not some mental image, but (typically) some opaque colored object in front of the eye, often at a great distance. One way of thinking about this is to analogize vision to touch; the eye reaches out to the object, somehow, scanning a distant surface in much the way the fingers of the blind can explore a texture. The epistemological "cut" between the objective and the subjective, between the object perceived and the means of perception, lies just at that interface, whose characteristics are thus apprehended. This ancient and entirely correct commonsense notion of vision can itself be geometrized as the "ray from the eye", and in principle can do all the work of geometrical optics as well as its physical complement, the light ray coming to the eye. It was the characteristic view of Plato, but also of Ptolemy's Almagest and the medieval Arabs. (See Vol.V., pp.37-41,55-59)

This latter, complementary view of light, represented by lumen, recognizes that light is emitted by physical sources, is reflected, scattered, absorbed, and thus the information obtained in vision is brought to the eye, which is now conceived as a passive receptor rather than an active probe. Holton's interest, in this essay, is to relate the duality in question to the development of Niels Bohr and his philosophy of quantum physics.

Rather incidentally, Holton traces the historical development of optics, in which this duality is so conspicuously and profoundly revealed (a standard reference is Vasco Ronchi, Optics, The Science of Vision, trans. E. Rosen, New York, 1957). Our interest has been to examine the apparently comparable development of thinking among our present-day adult students. It seems to us quite clear that the pedagogical-historical parallels in this case are both wide and deep, and merit far more extensive investigation. In the historical development of optics, one can see both the initial dominance of lux and the slow emergence of its complement, the physical conception of lumen.

A second and very suggestive discussion is found in Stephen Toulmin's The Philosophy of Science (New York, 1960). Toulmin choses geometrical optics as a relatively simple scientific thoery. His cental argument is that what is usually called scientific discovery is typically, on closer examination, what we have been discussing as category reconstruction - not so much new facts as new ways of conceptualizing.

4. The Hot and the Cold

A case history simpler in some ways than that of geometrical optics is that of a twelve-hour sequence devoted to an introductory study of heat. This also was a follow-up study of subject matter which we had tentatively explored on at least one previous occasion (see below, p. 38). Here again our interest was attracted by evidence that everyday experience and commonsense thinking involve a categorization of experience which is rather radically in conflict with that presupposed by a modern scientific account, even when the latter is reduced to its simplest terms. Here also, then, we might find an opportunity to study the teaching of elementary science in a way requiring of students a radical kind of category reconstruction.

Our first clue about the nature of this conflict and the conceptual difficulties which it may generate came from work with young children (ages 6-9), in which the study of heat and temperature occurred in the context of animals' and humans' protection against the cold. Any first-hand investigation which involves one's own temperature sense creates initial dissonance at the point where the perception of hot or cold depends not only on physical temperature but also on thermal conductivity. A stone floor is colder than a wooden floor of the same temperature if bare feet are the measuring instrument. A London bedroom feels colder than one in Phoenix when both are at the same temperature of 7°C. This difference has to do with the thermal conductivity of skin, clothing, etc. as affected by different degrees of moisture. For a different reason (which has to do with the heat of vaporization and rate of evaporation) the London 30°C. is hotter than that of Phoenix.

Indeed, a first-approximation account of the human temperature sense is that it genuinely is bipolar. It measures the rate of exchange of heat between the environment and the body, hot by degrees in one direction, cold in the other. It is thus not simply but only grossly related to physical temperature.

This is what our first and second graders told us. For example, orange juice cans of very hot and very cold water both got "warmer" over time, more slowly when wrapped in fur, faster otherwise. From both extremes the water approached the state of being near neutral, "warm." If we impute a mathematical model to the children, it is arithmetical rather than algebraic. When thermometers were introduced their interpretation of temperature was one of confusion; the thermometer went up or down, which seemed to have nothing to do with hot → warm ← cold.

The further experience we have had with adults appears to confirm the dominance of the hot-cold duality. Hotness and coldness appear, in all of the naive conceptualizations, as two polar opposites whose "balance" yields a neutral state near which we live and flourish; to be too far away from this state of balance is perilous. Lest this way of thinking be supposed to be merely some sort of naive error, we might remind ourselves that the same habit of thought has been basic to, and is still embedded in, our quite respectable notions of electric charge and magnetic polarity, not to mention many analogous notions of polarity and valence which we employ in the discussion of human affairs.

II, p52

In the present research, designed against the background sketched above, we sought initially to reinforce, rather than oppose, this perceptual polarization. This was a tactic, a pedagogical device, to bring freshly to mind ways of thought which we expected our teacher-students to exhibit and, because of the freshness of related experience, to be able to reflect upon.

A first exercise was to examine and feel a variety of objects having different thermal conductivity, but all in thermal equilibrium with the room we were in. The aim was simply to reinforce the validity, in its own terms, of our temperature sense. We did not actually measure the temperatures of these objects, which in a more painstaking investigation would have been appropriate (see Vol.V., pp. 253-260). In the course of our discussion we had ample confirmation of our teacher's lack of awareness

v. II, p. 15

of any distinction between thermometric temperature and our own biologic hot-cold indicators.

The second exercise was simply to mix very hot and very cold water in varying proportions and to construct a sequence or scale with as many steps as possible, in well-insulated cups, ranging from hottest to coldest. Thermometers were forbidden, so that only pairwise comparisons by fingers dipped in the water were possible. Our teachers thought the exercise rather trivial, as it turned out; the only surprises came when one's two hands (one previously warmed, the other cooled) gave contrary indications when dipped in the same water, and when different individuals made different judgments as to which mixture was perceived as neither warm nor cool.

On a previous occasion this same exercise had provoked high excitement and a strong commitment to quantitative comparison: Three of six teacher-students had expected that equal volumes of ice water and boiling water when mixed would be neutral or, at least, tepid and were surprised that this was not so. In both cases, however, the exercise was sufficient to consolidate the bipolar conception. As in Aristotle's account of the four elements, the hot and the cold are contrary attributes of things, substantive qualities which in suitable proportions will neutralize each other.

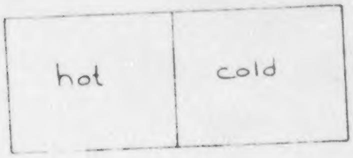
Our next exercise was again the mixing of hot and cold water, but this time using thermometers. Equal quantities of 80° and 10° gave a predicted 45° mixture. In a second exercise the hot and the cold were poured into a container divided by a thin metal barrier, and in a third case the thin metal barrier was a pleated one, having ten or twenty times the surface area of the straight barrier. In the second exercise the two batches of water, though physically separate, did in the course of a few minutes approach quite satisfactorily toward a common temperature (slightly below 45°). In the third case the approach to equilibrium was almost as rapid as when the batches of water were mixed directly with *no* barrier between them.

This whole exercise was intended, of course, to suggest or at least support another way of thinking about heat and cold; namely that one is dealing with something which can be said to *flow* from one place to another (in the second two cases from one part of a container to another, but which, unlike the hot or cold *water*, flows rather easily through a metal sheet. It had been our expectation that we would thus move on from the kind of Aristotelian, two-quality conception to something like the eighteenth century caloric theory: Heat could be thought of as a subtle, imponderable *fluid*. If this conception gained allegiance, then it would be possible to think of hot water as a kind of mixture of *water* and *heat*, the proportion of the latter to the former being *temperature*, a ratio such as calories per gram. Cold water would be simply a lower-temperature mix.

In one previous experience we have been amply rewarded: Below is an excerpt of the report we wrote:

> "In the first session we had talked about hot and cold as opposite qualities, about the hot and the cold as kinds of things -- what the old philosophers called "principles" or substantive qualities. The teachers' contribution, helped by some questioning of their experience, was pure Aristotle -- not a bad beginning and, we all now agreed, not far from the ingrained and reasonable patterns of common sense, including the common sense of most young ones.
>
> The history we presented was some Aristotle, with jumps to the thermometer, to Galileo, and to the origin of "temperature," which was a sixteenth century medical term signifying something also called the "temperament" of people, departures from a healthy state toward too much of "the hot" or "the cold."
>
> Along the way we had occasion to notice in ourselves another commonsense habit of thinking about the phenomena of

heating and cooling -- that heat is not just a quality but is something which flows from hotter to colder, some kind of fluid. This was inconclusive because one could also think of cold the same way -- cold penetrates. To underline this way of thinking, we went back to our earlier mixing of hot and cold, but now changing the equipment. We poured the hot and cold water into separate compartments in two plastic shoe boxes, one of which, a, was divided by a thin flat strip of aluminum and the other, b, by a much longer strip of pleated aluminum.



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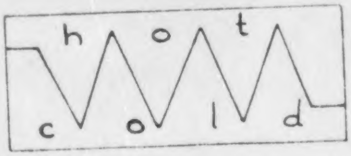

b

In both cases we got roughly the same result as when mixing hot and cold water directly -- after a while the temperatures in both compartments were nearly the same intermediate value we had found before, though the two batches of water could not now mix. In box a the process took a lot longer than when the hot and cold were mixed together, while in box b the intermediate value was reached almost as fast as with direct mixing.

In this context we found ourselves thinking about heat as a fluid -- it was no longer just the water that was hot -- but a second fluid, invisible and imponderable, which could permeate the tangible fluid water but which could also flow through metal, which water itself cannot do.

With the idea of heat as a fluid in mind we were near to a clear sorting out of the relation between heat and temperature as the physicists define it. A lot of heat in a little water would give a high temperature; a little heat in a lot of water would give a low temperature, even cold! Of course, there is still the possibility of two such fluids, a fluid of heat and a fluid of cold, just as there can be two sorts of electricity, positive and negative. We laughed about that; we weren't trying to prove anything but were just learning to think in new ways and to think about our thinking.

As we were all trying to think about this mysterious fluid -- it used to be called "caloric" in the heyday of its popularity -- three questions emerged, two of which Barry and I were prepared for but one of which astonished and delighted us. This question was "Does the idea of heat as a fluid mean that the amount of heat is finite?" If heat is a fluid which can be present in things in different concentrations, and if

> cold is only a relative absence of it -- not a positive something in its own right -- then you could imagine bleeding all the heat out of something os it couldn't get any colder. Suddenly our new way of thinking had shown logical power; it had suggested something genuinely beyond the range of commonsense experience, an absolute of coldness, an "absolute zero" of temperature. Is there also an absolute of hotness? Not in this way of thinking, at any rate. Everyone had heard that mysterious phrase "absolute zero." Suddenly it came alive and we could meet over it as equals.
>
> Other teachers immediately thought of another beautiful, related topic, the fact that the earth is a reservoir of solar energy flowing in from the sun and out again into the darkness of space. We had only touched on radiation as a flow of heat, but again the key was conservation. We weren't ready yet for the grand formulation, the conservation of energy, but the idea was there."*

In the course reported here the result was quite different, a humbling reminder of an inherent variability in situations of teaching-learning. Now, however, we caught an important nuance which occupied us for some time. There was serious objection, indeed rejection, of the notion -- metaphor -- that heat flows, or that, in consequence, it can be at all called a fluid. (See Vol.III, pp.239-245) When this objection was probed in terms of other examples of heat (or cold) getting from one place to another, the acceptable language turned out to be that heat is conducted from one place to another. In the instructor's mind, the two notions were equivalent (heat flows through a conductor, as does electric charge; the water pipe or gas pipe or wire is a conduit). But in our teachers' reactions we sensed a genuine divergence of ideas. That which conducts heat (stove top, a pot, etc.) does so not simply as a passive medium through which heat flows, but as an agent which transports the heat from A to B.* This contrast also applied to materials which conduct electricity. We were not able to probe more deeply for the source of this divergence. But it was clear that a physicist's metaphoric formulation such as "heat never flows from cold to cold" would cause just as much trouble as "light travels in straight lines."

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* From "Getting into Subject Matter, a Missing Ingrediant in Education", Outlook, #24, Summer 1977, pp.30-31. Mountain View Publishing Co., University of Colorado, Boulder, CO. 80309.

* Reference: III, 238-246.

In both cases it is not the intended content of the statement which causes confusion, but the patent unsuitability of the commonsense content of flow and travel. A conductor conducts, transfers, and the verb is an active verb, needed perhaps because one is aware of no holes through which heat -- or electricity -- might be said to flow.

I wish here to insert a rather speculative interpretation, which we have not been able to follow up adequately. If this interpretation is right we will find ourselves dealing with a superficially unscientific scheme of thought which has more commonsense merit than meets the eye.

To talk about the flow of heat (or electricity) imparts, from all the commonsense analogies of flow and fluid, the notion of empty spaces, holes, channels, through which a fluid could flow. However, there is a rough and general relationship which contradicts this notion: the best conductors of heat are the most obviously dense, or pore-free materials, such as metals. Glass conducts, fiberglass insulates. Moreover, what is transmitted is not any ordinary sort of material thing, but something which we will in the end call energy, something passed on by the material components of the conductor, but not these components themselves.

In other words, the commonsense notion of the conductor as agent implies an interaction (between heat and matter) which is closer to reality than the very thin abstraction of heat "fluid", which we had aimed at and which has the sole and primary virtue that it implies a quantitative conservation law.

In the present class, unlike the one referred to in the quotation above, the conception of heat as conserved, hence as sometimes "latent" (and thus finally as a form of energy) brought to our attention no very dramatic evidence of conceptual reconstruction. Instead we got into a whole range of questions about the forms of energy, and experienced the overload discussed above (p. 16): this is particularly clear in our own staff discussions with Maja, who is voicing problems she shares with teachers in the class (see Vol.III, p.250, for a question about heat energy and cold energy).

I have spoken before of the fact that conceptual reconstruction involves repeated practice; it is of the nature of habit. After our extensive discussions of heat, heat flow, or heat conduction, we gave a fairly conventional liquid nitrogen demonstration (see p. (290) et. seq and staff discussion, p. (320) et. seq), which released far more discussion than we can analyze here. What is central, we believe, is the excited recognition of a temperature world colder than daily experience suggests, or than our senses can probe. The peak moment, in our discussion, came after the intense boiling of the nitrogen, occasioned by a bit of ice thrown in. The ice was hot!

In the previous class (see page 39) the theoretical point had emerged that if one thought of heat as some kind of fluid, then this carried with it the implication that there was an absolute zero of temperature. In our more recent class, this notion of a heat fluid had been rather firmly rejected, and the realization that something very cold (ice) could be much hotter than something else (liquid nitrogen) was still startling, though this fact was already implied in the general acceptance that "cold" is equivalent to a "relative absence of heat." Absolute zero came along, as an idea, in a differnet context (see Vol.III, p.277-281). The two-attribute conception dies hard.

5. Size and Scale

1. Area

At first sight it is puzzling that the concept of area appears, among a large majority of adults we have worked with, to be very poorly grasped. The confusions around this concept, as we have observed them, have partly to do with the arithemetic, but lie perhaps more essentially in the relation between the intuitive notion of the expanse of a surface and the metrical notion in which this expanse gets represented by some number of units.

We can illustrate this difficulty from past experience and from our recent college course (see below). From past experience: "How can we find the area of our footprints?" "You can't, it's not a rectangle" or, "Wouldn't it do just to measure around it?" This latter confusion of area and perimeter has been often observed, by ourselves and others, for example in math workshops for elementary school teachers. It suggests that there is no clear notion of dimensionality. Going a step further, we find extensive confusion regarding metric relations of areas and linear dimensions. In its most rudementary form, this appears as the supposition, for example, that two miles square is the same as two square miles, or that the area of a football field is (perhaps) 150 square yards or (perhaps) 200 square yards. In our energy course, this behavior was exhibited by many, and always by at least one student in every mention of the topic, throughout the course. It was a persistent trouble, in other words, which did not yield easily to normal instruction even when, as in this case, we anticipated the trouble and gave it considerable attention in class. A second example, involving discussions of the cross-section area of the earth exposed to sunlight (or of a coffee can held in various positions), made it amply clear that there is persistent difficulty in understanding the range of contexts and applications across which this concept of area has meaning and utility.

However easy and familiar the concept of area may seem to those who understand it, our records imply that in a wider population it is a very weakly developed concept. As scientifically naive people grasp the notion, moreover, it appears to suffer from pedagogenic troubles. Areas have always been represented on paper or blackboard as bounded, flat surfaces, rectangular or triangular. Such notions as the surface area of a sphere (or even of a cube!), of lung or stomach tissue, or the cross-sectional area of a tree trunk or leg bone, seem often to be inaccessible. The persistence of confusion with area units (square inches, inches square, for example) suggests for teachers first, that arbitrary units of area (not labeled by an L^2 label) should be used transitionally, and second that ways should be found to work or play with surfaces having different shapes but the same area.

cf III, pp 73-82

Finally, and most inaccessibly, we find the deepest trouble to lie just where the concept of area has its greatest scientific utility, in the grasp and appreciation of similitude, the fact that the areas of similar shapes (whatever the shape!) are in the proportion of L^2, the squares of any corresponding linear measures L.

2. Volume

The existence of difficulties with the concept of volume (analogous to those with area) may be concealed by the familiarity of common volumetric measures. On the other hand, the use of these measures also links volume closely to mass, often so closely that the distinction cf III, 122-125

between them is unavailable; whereupon density becomes a mystery.

Some of our observed difficulties center about the use of L^3 measures -- cubic yards or cubic inches -- and this appears to confirm our interpretation of the similar difficulties with L^2 measures of area. In the minds of most of our teachers, the exercise of determining the number of unit (wooden) cubes in a large cube of 2, 3, 4, ... linear units was totally novel and its results immensely surprising (as was also that of finding the successive surface areas). The notion that this counting of unit cubes, gives a measure of volume, and exposed faces, gives surface area, appeared to be very surprising and not immediately assimilable. We developed the conjecture that there are two relatively common disjoined notions of what we would call volume: of capacity (the empty quart jar, measuring cup, fifty gallon drum) and of bulk, the latter related to, and not sharply dissociated from, "amount of stuff."

MA. The Teaching View, pp 31-2 & v. III, pp 144-146

3. Scaling

Our aim, in this short course, was to introduce teachers to the ways in which the qualitative features of natural phenomena vary with size. To this end, we introduced some simple investigations of surface forces -- drops, bubbles, foams. We talked about Galileo's scaling of the beam, and had the students read the famous Julian Huxley article "On the Size of Living Things", and tried to confront them with diverse other phenomena of scale change, such as cooking times and melting times, the changes in the properties of cream or soap solution when bubble size is reduced by continued beating, transforming a liquid into something with quasi-solid properties.

Once again, we aimed to begin with relatively complex phenomena, retreating to simpler ones in the effort to understand. It is that retreat which led us back to the topics of area and volume, and to the recognition, once again, that the reconstruction of the categories of stored experience, when aimed at scientific generality, involved a figural-formal transformation (see ref., p.10) which is far more radical than is usually realized or acknowledged. For that reason, the discussion of Maja Apelman, written from the point of view of her own close empathetic perceptions, should be examined carefully in connection with the present topic. Since much of this needed reconstruction belongs to a general examination of the role of elementary geometry in this figural-formal reconstruction, one should also examine the discussion of Ronald Colton (Problems with Mathematics).

A simple (and still quite conjectural) model of our students' initial perceptions of scale phenomena would suggest, first of all, that there is a cluster of commonsense concepts relating to the topic, dominated by a central scale not unlike that of hot-cold, a scheme of contraries with something human-size as neutral. Size, like temperature, is **then not a** purely relative matter. Some things are intrinsically small, others intrinsically large. So to make the model we chose a unit, something like a meter for length or kilogram for mass. It could as well be a millimeter and gram -- no matter. We then go in both directions, using some measure for small and large. It might be a characteristic length or distance, an area, a volume, a mass. More probably it might be some weighted average of length, area, volume, mass... In an ecologically random sample of the things we encounter and take account of in the course of daily life, these variables are all rather strongly correlated with each other, so it matters little what weights we assign to these incommensurable dimensions --except in extreme and unusual cases, which common sense typically ignores-- in modeling the notions of large and small. It follows that if this model is at all correct we must be prepared for students who have great difficulty in distinguishing between area and perimeter (for example) simply because these usually go together and you don't have to notice how essentially different they are.

It also follows, pedagogically, that we should subject our students to the unusual cases, the cases which are ecologically abnormal: thus a sequence of shapes, having constant area, which step by step have perimeters which approach infinity. Can you equally well have shapes of constant perimeter whose areas increase beyond bounds? This is abnormal pedagogy, but perhaps important. What applies to area and linear dimension can also apply to the real-word, 3-D case as well. The Gibbs ink-drop is a constant volume of ink with endlessly increasing interfacial area between ink and water. It corresponds to the abstraction, still important in hydrodynamics, of the incompressible fluid. There are natural systems like the root or leaf system of plants, the circulatory system of animals, of vast surface area per unit volume. But these escape notice.

To reconstruct our intuitive, figural category scheme in such a way as to explicate the three metrical concepts of length, area, volume, and the relations among these, we are again committed to replace a figural, common sense structure by a formal construction involving a network of precise geometrical concepts. In the corresponding case of optics the central stumbling block appeared in the development of and reliance on a formal geometrical abstraction, reconstructing all prior ideas of light and vision in terms of it; the light ray. In the present case the figural ⟷ formal transformation* involves the re-presentation and transformation of our intuitive grasp of "size", including some intuitive awareness of span, surface, bulk. The elements which formalize these aspects now involve the recongition of three distinct additive measures and of their geometrical relations to each other, measures which, moreover, are conceived in abstraction from all other properties of physical objects including color, mass, and even shape.

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* See Jean Bamberger and Donald Schön, "The Figural ⟷ Formal Transaction, DSRE Working Paper WP-1, MIT Room 20C-124, June 1978.

To come then, finally, to grasp the central importance of such a ratio as surface/volume or volume/surface, and to conceive this doubly-abstract notion as richly indexed to a great variety of natural objects and phenomena in the physical and biological world, is to develop a return pathway from the formal geometry of measure to an enriched perception of the figural, the concrete objects of experiences. The volume/surface ratio for roasts or loaves of bread of different sizes will directly relate to cooking times. The ratio is equivalent to the average depth to which heat must penetrate, a theorem seldom understood. The topic leads on. into the wide world of physics and the complexities of biological adaptation. As a tour de force George Gamov compared the metabolic rates of a mouse and a star, and showed that these were in direct proportion to the respective surface/volume ratios. In much of biology the problem appears in a different form, as a problem of adaptation, thus as to how the elephant can manage to have about the same ratio of gut area or lung area to weight (or volume) as the mouse.

In the case of the present history, perhaps more than in the case of our other experiences in these seminars, we see the sheer fact of very major difficulties, very slowly overcome, as our contribution, with only the conjectural model outlined above to explain it. It is therefore appropriate that the entire essay of Maja Apelman (see above) draws upon the detailed record of, and upon her reflections of personal experience in, this very subject matter. It is also appropriate that in his essay, on mathematical difficulties, "Problems With Mathematics", Ronald Colton has also examined the topic of size and scale in relation to other geometrical and arithmetical abstractions which we know to be essential to the scientific reconstruction of the categories of nature-knowledge.

Here I should like to add a specific emphasis, however, which I believe may lead to some further understanding of the difficulties with size and scale. Robert Davis has postulated a major source of difficulty in early arithmetic. Much of Davis' discussion of frames* is appropriate to this commentary. Very briefly, Davis proposes a theoretical scheme of "frames". These are, for all present purposes, equivalent to the categorial taxa, "file folders" which we have referred to in the explication of William James' basic account of human understanding and reasoning.

v III, pp116 -117

In Davis' account of early learning of school mathematics he proposes a hierarchical sequence of frames, from earlier-learned to later, in which an inadequate learning or differentiation of later frames explains errors which, typically, are not random. They predictably involve some regression to the use of frames learned earlier (and better), from which they have been inadequately differentiated. Thus in the first learning of subtraction children tend to identify the demanded operation with the already consolidated addition frame. In Jamesian language their "sagacity" is to recognize the demand for a binary operation, but not to differentiate the cue of "add" from that of "subtract". Having first learned only one binary operation, addition, children have also learned -- inappropriately -- to ignore the "+" sign; it is always implied in the early months of schooling, and therefore, in that context, is redundant information.

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* Building a Metaphoric Theory of Mathematical Thought: The Postulation of Certain Specific, Explicit, Commonly Shared Frames; by Robert B. Davis, Journal of Mathematical Behavior, Autumn 1980, Vol.3, Number 1, pp.187-201.

In a similar way multiplication, inadequately understood, can also regress to this "primary grade undifferentiated Binary Operation Frame". Hence, the area of a football field is 150 square(?) yards.

But even higher up the standard curricular ladder, and of special interest in the context of L, L^2, and L^3 is what Davis calls the Label or Unit Frame. In the descriptive use of numbers there is always implied a label (class name) or unit. This is overlooked in the learned habit of manipulating numbers in the abstract (as in most school arithmetic and algebra). It is therefore plausible to suppose that there can be a regression from adequate understanding of geometrical (or other) measures, to the use of a frame which involves only pure numbers and thus loses the geometrical essentials.

We noticed such an effect in our work with wooden cubes. Our teacher-students built successively larger cubes from unit ones, and became deeply involved in the number patterns, thus:

edge-lengths	outside surface	number of unit cubes
1	6	1
2	24	8
3	54	27
-	-	-
-	-	-

It then appeared, however, that in the excitement of discovering that each row is of the form n, $6n^2$, n^3, they lost the sense of surface area and volume this represented. For example, the subdivision of a unit cube appeared as a totally new problem.

A very special cue appeared recently in a discussion with one of our staff, Maja Apelman, who often is close enough to our students' difficulties in her own thinking to suggest the nature of theirs. She had been thinking about a cube 3" x 3" x 3" as having a volume of 27 cubic inches which could therefore be rearranged (as a box shape) in many ways. One suggested was 3 x 9 x 1; but then, she said, multiplying by 1 "doesn't change anything", so it would give only square feet, not cubic feet! Very clearly there is regression here, from the frame for geometrical measure to that for pure arithmetic.

The suggestion emerges, very strongly, that although difficulties of this kind do involve some inherent critical barriers, in making the transition from commonsense qualitative conceptions of size to a more orginized geometrical understanding, yet additional difficulties are created by the artificial separation and linear ordering, in our curricular practice, of interconnected concepts and operations. It would be a modest beginning if we taught addition and subtraction always together, or multiplication and division; possibly even all four. More widely still, devised a style for the spiral curriculum in which these artificial separations were only briefly required and always subordinated to the evolution and strengthening of the conceptual network which constitutes intelligible mathematics.

Mountain View Center

Research on Critical Barriers,

Fall 1980 - Spring 1982

CRITICAL BARRIERS: THE STUDENTS' VIEWS

by Maja Apelman

Introduction

In this essay I shall try to describe how one of the topics of the first teachers' seminar (Spring 1981) was taught and how it was experienced by the teachers who were our students. I am making use of the following sources of data: weekly notes written by teachers; my notes, written during this seminar and during two previous Mountain View classes on the same topic; transcripts of classes and of staff discussions; interviews with teachers during and after the seminar; and informal talks with staff before and after each class. I hope to convey a feeling for the atmosphere of the seminar and to discuss how the teachers and occasionally the instructors reacted to what happened in class.

In the Mountain View setting and with instructors I knew well, I was able to fulfill my role as student advocate and liaison person between project staff and participating teachers. As far as learning new and difficult material was concerned, the teacher-students regarded me as one of them and, in addition to sharing frustrations and discoveries, we had some good laughs together about concepts which we knew we were supposed to be "getting" but which kept eluding us. By having continuous access to the instructors -- David Hawkins and Ron Colton -- I was able to communicate both my own and the teachers' thoughts and feelings about the content, structure, and pace of the classes. Sally, one of the teachers, was hired as a part-time assistant. She attended the weekly staff discussions, purchased materials, and helped with preparations. I found her presence at the staff meetings extremely helpful since hers was a second voice that spoke up for participants' troubles.

For the second teachers' seminar, there were many planning discussions which preceeded the selection of the topic Light and Color. I knew what the goal was even though my understanding was limited. But before this first class, I only knew that size and scale was one of David's favorite topics, and when some teachers asked me what the course was going to be about, I told them that the topic Size and Scale in Nature was central to the understanding of many important things in the real world but that I was taking this mostly on faith: I knew it was important because I trusted David. Notes I had taken in a class on size and scale many years before touched on some of these connections to the natural world but apparently I hadn't been ready to make the connections myself and therefore did not remember them. I was surprised to find these old references when I reread the notes in preparation for writing this essay.

"Size and Scale is a pseudo-structure for the seminar," David said to me. "If you have length, area, and volume in place, you have a useful ladder to understanding things from atoms to galaxies." I was looking forward to gaining some of this deeper understanding.

First Meeting: Soap Bubbles

The first meeting with the teachers was not planned as a regular class. Some people had yet to make their final decision to attend -- they wanted to hear more about the research itself, as well as to clarify such matters as the duration of the course, district in-service credit, etc. David started right in with the main topic of the course. He asked, why do people have trouble with what surface area means?

> This is a very common source of intellectual difficulty, and not just because of the mathematics. There seems to be a situation where you don't have to do much computation and still you get into trouble. Why would that be? What is there about ordinary, everyday human experience that would make it difficult to disentangle? Area, surface, it's around us all the time. It is the sort of thing that intrigues me, because it doesn't seem plausible that it should be that difficult...If people don't understand something, it's not because they are stupid, it's because they have preconceptions which they have learned, and often learned under conditions which make these preconceptions useable and useful. Then we require that people give them up. It's like giving up a friend.

He explained further that his interest was in students' difficulties in finding out how a person was thinking, in the so-called naive questions people might ask. "Imagine being valued for that!" Hedy, one of the teachers, commented. In a few short minutes, the tone of the class was established. Hedy probably voiced the feelings of most of the teachers present, and David was convincing in communicating to the teachers that he was deeply interested in their individual ways of thinking, of learning, and ultimately of understanding. "This is the complete reverse of what is ordinarily valued and it's one of the main reasons why these fascinating naive questions have never been seriously looked at," he said.

After some more discussion about teaching, learning, and research, Mary wanted to know how we would go about getting past critical barriers once they had been identified. "That implies that we look at specific topics and get into the substance of a scientific explanation," David answered. "Let me give you an example." He got an egg beater and a bowl of soapy water, prepared earlier for use during the class.

We started beating the soapy water, watching bubbles form, multiply, diminish in size. The whole mixture grew in volume while air was being beaten into it, but when we continued beating, it got stiffer. David drew our attention to the change in the mix:

> "There is an awful lot more internal surface of little bubbles, there are more and more little bubbles and if you could measure the total amount of surface where there is liquid and air in contact with each other, it increases and increases and increases. There is a big change in the ratio of volume of water to the surface area that the water has in common with air...and the bigger you make the surface per unit volume, the stiffer the stuff gets."

I now wonder whether any of us knew then what David was talking about. Hedy wanted to know if there was more "surface air" and I said that I was so mixed up that I didn't know anymore "what's water, what's air and what's soap." David explained that the beaten-up solution was a mix of a liquid and a gas that acted as a solid, adding that we had to start thinking about what it was that made this difference. "It is a problem related to just the amount of liquid film that is in there."

Still puzzled, I asked for a parallel example that might help me to understand what happened to the soapy water. Ron mentioned water drops, comparing big drops with the tiny drops found in clouds. "The big drop of water doesn't have any stability." What is a stable water drop? I wondered. "Does stability mean not breaking apart or not being real strong?" Instead of an answer, I got a promise that we would play with water drops on wax paper in the following class so we could study and observe how different-sized drops looked and behaved. I don't know why the concept of stability gave me so much trouble, but it came up again and again during the early classes and staff discussions.

Soap bubbles were one real-world example with which David and Ron wanted to introduce the topic of the surface area-volume relationship. Ron had also brought in a dead plant with a root, pulled up from his garden. He pointed to all the little roots and fibers at the end of the root stressing the enormous amount of surface in contact with the soil and the water. We talked about what happens when you transplant a plant and cut off the tip of the root which has most of the fine root hair. Sally found this example helpful, "but those bubbles," she said, "I just have to take your word for it, I don't picture it at all." I I didn't understand the bubbles either, but the surface area of the root hairs presented me with another problem. "I don't ever think of roots as having a surface." I said. "I think of their length but not about their surface area. I don't look at these roots and think: if I put them all together they will make so much surface area." "But every tiny bit of water has to go from some place in the soil across the surface of the root to get inside the plant," David said. This is so obvious when you know and understand it. I think it was hard for David to believe that one can enjoy gardening and care for house plants and never wonder how the water gets from the soil into the plant. I knew that the roots transported the water but I had not thought about the significance of their surface area.

In fact the term surface area itself gave me problems. When I asked if other teachers had trouble thinking of an area as being made up of "long strings of things" (intestines had been mentioned, the elephant's huge gut), Mary said she wasn't sure she had trouble with that but she wondered why she should care about it. "Why is it important?"

I asked finally if we could just talk about surface and leave out the word area which seemed to cause my confusion. David suggested we talk about "the amount of surface." I liked his acceptance of my problem and his willingness to change the terminology. This is another way of validating a student's thinking.

Since this was our first meeting, most of the teachers weren't ready to raise questions as freely as I was. But asking questions and voicing my confusions were what I was expected to do. My questions were always genuine. The teachers sensed that and gradually they became freer in exposing their own troubles.

We did not ask for "homework" that first week, but two teachers, Hedy and Sally, brought back some notes. Sally wrote:

> ...How do they know that the mixture becomes more solid because the surface area of the bubbles becomes greater, thus creating more strength? And how do they know that increased surface area in relationship to the volume creates strength?... I'm taking someone's word for it as I always have in science. Could you

> actually see the surface of these bubbles through a microscope and analyze the relationship between surface area and volume? Where is the starting point for this understanding?

"How do they know..." expresses a feeling common to nonscientists. Scientists' knowledge is far beyond our understanding. We have never known how to gain entry into their world because that entry has been offered only on the scientists' terms, which have been incomprehensible. So we remain outside, wondering, "How do they know?"

Hedy had made a list - "off the top of my head" - of all the things that seemed problematic to her. It was a kind of confessional, alerting the staff to some of her troubles:

- estimating volume: I'm aware of this when I try to decide which size container to put left-overs in. I invariably choose containers that are too large...
- diagrams - yeach -- especially verbal descriptions of spatial relations. I have a physical desire to crawl beneath a table.
- Percentages are almost incomprehensible to me. That dates from 8th grade.
- geometric formulas - utterly meaningless. Square roots, light years, same.
- Light spectrum and photography I have never grasped. Many concepts of physics and astronomy are interesting to me. I enjoy hearing the explanations but I never retain any of it. It's like a fairy tale only I can't remember how it goes.

Water Drops

On the agenda for this class were: playing with drops of water -- wax paper, eye droppers, and food coloring were set out for that purpose; experimenting some more with beating soap froth as well as egg whites and whipping crean; observing how oil behaves when dropped into water; and floating needles and razor blades. I didn't know what all of that had to do with size and scale but I didn't really care.

David's favorite way of preparing for a class is to experiment with the materials he wants to use, and so I got a chance to play around with the razor blades and needles, learning that the needle will float if it doesn't get wet, which sounded strange to me, and that if you rubbed it against your skin, it will get oily and float more easily. "The insects which walk on water keep their feet dry," David said. Do insects have oil on their feet? I wondered. I now know that there is a delicate balance between the pressure of the water spider's foot and the surface tension of the water on which the little insect puts its weight. But at the time of the course, the idea of the water's elastic skin was not yet part of my mental framework. I am amused by the numerous question marks all over my last year's notes. It's nice to look back at your own confusions when they have become resolved.

We had decided to start the class with the planned activities. Everyone began by playing with the water drops on wax paper. "My first reaction was surprise that such simple materials could be so interesting," Sandy wrote in her notes; "The investigation was so open-ended and non-directed." There are many careful observations of water drops in the teachers' notes. Ann wrote, "It's fun to watch the small bubbles be 'gulped up' by larger bubbles. The drops seem to pull together. The small bubbles seem more round and almost have a peak compared to the larger bubbles." Hedy, whose initial reaction to our set-up was negative -- "I won't like it" -- nevertheless allowed herself to get involved. "I am amazed by how self-contained each bit of water is. I am delighted by the movement of the drops. As I pull them around they change shape - amoeba-like. They have charm..." I, too, found the water drops fascinating as they slithered over the paper without leaving any sort of impression. "How come the water isn't wetting the surface it is on?" I wondered. "Well, that must be the non-mixing of wax (oil) and water. So they really do not mix." Personal observation is more convincing than a teacher's explanation.

At one point David filled an eyedropper with soapy water and announced that he had a "mystery liquid": just the slightest trace of that liquid on the surface of a water drop made it disintegrate. That raised a question in Hedy's mind: "If soap makes water strong enough to hold a bubble shape, why does it make water drops on the paper lose their shape?" That apparent paradox bothered me, too. How could soap destroy a waterdrop but strengthen a water bubble?

After a while, some teachers moved to another part of the room to beat the various liquids we had prepared and to experiment with the oil and the floating of needles and razor blades. Others remained with their water drops, continuing their observations. Beating soap froth prompted Mary to write: "I wonder about mixing air with liquids. Since air is invisible, I have difficulty imagining what must be actually happening." Mary found it hard to think of "invisible" air. I thought of air as a kind of nothingness that fills the spaces between real objects. During one conversation, David had described air as having molecules with space in between. I found that amazing. How can there be space in between when I think of air as nothing to begin with? My nothing view of air was a real stumbling block to my understanding of soap froth. The millions of little interfaces which increase the internal surface area and thus strengthen the mix troubled me for quite a while. No wonder! I thought of "real air" which interacted with the soap film as surrounding the entire soap froth, but the inside air had apparently remained in my nothing category.

The teachers who had remained with the water drops became fascinated by two discoveries: the magnification of the print on newspapers with which we had covered the tables and an inverted reflection of a large skylight in the tiny waterdrops. Both Sally and Sandy wrote about the skylight, and both added an interesting comment about their feelings as students of science. Sally wrote:

> I was most fascinated with the way the drops magnified anything under them and the way a bubble in a drop demagnified anything under it. We discussed magnification with a convex lens and demagnification with a concave lens. That seemed clear enough. Is it really that simple?
>
> The rest of the time we looked at different objects through various lenses. Print became inverted as we pulled the magnifyer away from it. But I was puzzled about the skylight: it was always inverted. Why? I went home wanting an answer (we had had so many questions and so few answers) and so I looked in the encyclopedia. I think I found a partial answer: "An object being examined through a magnifying glass is always kept at a distance from the lens that is less than the focal point. If the object is at a distance greater than the focal point of the lens, an inverted image is found." I assume that our lens was always at a greater distance from the ceiling than its focal point, thus the image was always inverted. So what is the focal point?
>
> It's exciting to think I might have observed something and learned from the observations. But after all that looking and questioning, why is it that I wonder if I've drawn any correct conclusions? When I see a math pattern emerging, I'm excited and confident about continuing it. I know I'm on the right track, but in science I don't have the background to know if what I think makes sense.

Sandy also wrote about her discovery of the inverted skylight in the water drop. But it wasn't the inversion that intrigued her. She wondered "how such a large area as the ceiling could be contained on

> such a small surface as a water drop. I was shocked and perplexed at this phenomenon and dismayed by the fact that no one else was equally astonished...How quickly I abandoned that track of thought when it did not meet with shared interest; or, when I sensed that it might be the least bit 'obvious.'"

It's too bad Sandy did not pursue her interest. But in a class where people are encouraged to go in their own directions, one student's need for feedback may come at a time when others are too involved with their own observations to respond. If Sandy could have brought her question to David's or Ron's attention she would have received a different reaction.

Hedy wrote of her feelings when she asked David a question about the change in the water drop's surface tension when soap was added. I don't know what David said to her but this is what Hedy felt: "That was not what he wanted me to be thinking about... I just had some of that older feeling of I'm not doing it right, whatever it is...I've got to go where he's going and I don't know where that is."

These were intelligent women and successful experienced teachers, yet in the unfamiliar domain of science, old insecurities, fears, and frustrations quickly arose. One couldn't imagine a more relaxed, more supportive setting than the one we tried to create for these meetings, but old feelings are strong and it didn't take much to bring them out.

I would like to quote from one more student's paper. Shelley gives a wonderful description of her gradually growing interest and involvement in the water drops and her ultimate fatigue after a period of hard thinking.

> I started out with very random intentions -- joining bubbles, pulling, blowing, etc. -- feeling like I didn't really know what I was supposed to be doing. I quickly became fascinated by the effortless gliding of the bubbles over the paper - it was a wonderfully quiet and relaxing activity. I thought -- this would be great after an intense, thought-provoking activity. BUT before long I began noticing patterns - the large bubbles pulled in the small ones; the smaller the bubble the more spherical; I could blow air into the bubble making a bubble within a bubble (what is a bubble?); the bubbles seemed to magnify slightly, but a small bubble inside acted just the opposite. These patterns began to give direction to my randomness and it didn't matter what I was supposed to be doing. I began sharing my discoveries with those around me and I became fascinated with the magnifying and "demagnifying" powers of the bubbles - like looking thru both ends of binoculars. The little air bubbles within the larger bubble had pushed the water aside making a concave surface - opposite of the water's convex surface.
>
> My quiet, relaxed mood was gone and I wanted to find out more about lenses. But I didn't know where to go from here. Viewing print thru the double lens made things backwards and upside down. I don't need the answers to all this because I know I'd never remember them at this point but I seem to need some directions so that I can go further or be sure my observations are correct.

> All of these things kept me so absorbed that I didn't want to join the others doing something else. I couldn't handle any more ideas!... I left feeling stimulated but a bit drained and frustrated too. I feel like I'm at a standstill and there is so much to learn.

Shelly expresses the frustration experienced by many adults who start to study science after avoiding it most of their lives. The feeling of "there is so much to learn" can be overwhelming, especially when you work with David who believes in starting with broad complexities rather than with premeasured little units which can be memorized but which rarely take you to the larger more exciting understanding of science.

Since there had been no time for a general discussion at the end of the first class, we had decided to start the next class with a group meeting so teachers could ask questions or share experiences. David opened the discussion by expressing his hope that the teachers hadn't been thinking "I wonder why we are doing this," but had been able to get involved with the materials. The teachers were involved, some more so than others, of course, but I'm sure they were also wondering about the purpose of the activities. Harriet, for example, ended a detailed account of her work with drops and bubbles with this question; "What to do now with this information?"

David knew why he wanted us to play with drops: it was another illustration of the surface-area-to-volume relationship. A small drop has a much larger surface area in relation to its volume, so the surface tension of the drop -- the tendency of the skin around the drop to pull together, to shrink into the shape of a sphere -- was stronger than the gravitational force which pulls the water down. A large water drop, with less surface area in relation to its volume, was more affected by gravity and therefore tended to flatten out.

This very important relationship of surface area to volume, on which the whole topic of size and scale rests, was totally lost to the class. David intended to use it as a starting point for numerous excursions into the world of nature where this relationship plays such a central role, but as the transcripts of the following classes will show, we hardly got off the ground. Interesting, open-ended activities had been planned for the first class to introduce the concept of the area/volume relationship. The teachers enjoyed the activities but they also got interested in many non-planned topics. Marsha, for example, said that she had tried to whip the vegetable oil - with no success. Why would cream whip, and not oil?

Here is David's answer:

> There are lots of phenomena that you can observe and get interested in...what do you do with kids when they ask why questions, like the child asking where living things really came from? You say to yourself: "Oh Lord, the words that I would use would not be understood," and you try to direct their attention back to more things they can learn at their level. You probably evade their questions. I can tell you right now I haven't the vaguest idea what sort of answer to give to that. I think I could begin to evade it somewhat. Could you whip butter into foamy stuff? Butter is the fat, cream is a mixture of other things. I can see a difference between cream and oil.

David's "evasions" sometimes annoy me but I understand why he is doing it. He wants to direct the attention of learners (of all ages) back to the phenomena under investigation, back to the "things they can learn at their own level" He encourages further observation and experimentation and gives just enough guidance to make progress possible. He doesn't like to supply answers which he thinks won't be understood. He prefers the more circuitous, more time-consuming route that learners have to take in order to make their own discoveries.

We had a long discussion about cream, mayonnaise, salad dressing, homogenizing, and suspensions. Then David tried to get back to his topic: "Between the water and oil there is a little surface layer: a drop of oil floating on the water is like a bubble of air." He talked about the skin of the water which won't allow it to mix with oil, and about the reflecting layer where oil is floating on water. When he was finished, Ann asked: "I don't understand why the images are upside down." Referring to the choices teachers have to make in school when children become interested in things that aren't part of the "planned lesson", David asked that all questions on magnification and demagnification be temporarily suspended.

Hedy said she had a question about bubbles, the same question she had asked in her notes: if soap makes better bubbles, if it strengthens the bubble shape, then why would it make water drops collapse? This was another why question which David did not answer directly. He repeated Hedy's observation, adding more descriptive details, and then pointed out an apparent paradox: "The soap makes the surface weaker, the drop doesn't hold together as well. On the other hand, you can't make bubbles with ordinary water that will last whereas if you put a little soap in the water, they will last. If you keep the humidity high so the bubble can't evaporate, the bubble can last for hours. Soap bubbles are stable although their surface is practically weaker than it is on water." Instead of explaining how that can be, David mentioned similar examples of apparent contradictions -- heavy cream which is thick, yet light (that came up earlier in the discussion), and children's use of the words big and little or fast and slow to describe a variety of different attributes. This led to an interesting discussion about language and how labeling different properties with the same word can get you into difficulties in understanding those properties. I don't know whether Hedy felt that her question was answered but she seemed perfectly satisfied.

Mary now had a question. She wanted to know if the surface tension around the water was a separate substance from the inside of the drop?

David acknowledged that the outer surface looks like a different substance but explained that it isn't: "It just acts differently. When it's down in the interior it acts one way and when it's out where half of its neighbors are not there, it acts in another way...It's the same substance but at a boundary it behaves differently from the way it behaves inside."

Mary surprised quite a few of us with her next question: "Isn't it that way because of the hydrogen bonding?"

David agreed that it has to do with the way water molecules interact with each other but he added: "Let's stay out of the molecular world for just awhile because we want to stay on a more elementary, child's level." Rather than enlarging on the role of the molecules, he talked about the problem of teaching young children about atoms and molecules.

Mary, however, wouldn't let go. As a teacher, she says, she wants to have the understanding of the bonds. "I couldn't imagine that the air would make the surface that way, but that instead of those bonds reaching up to other water particles, they are reaching around."

"That's a good hunch," said David. "Water molecules have bonds, they reach out and hold on to each other. At the surface they reach out above and since there is nothing for them to grab, they may have to reach out in a different way and that may be what makes this surface layer stronger or create different surface properties."

David, however, emphasized that this kind of explanation would not be meaningful to elementary school children. Even with us, he would rather stay away from molecules and concentrate more on observation. That suited Hedy fine. "I really don't care why drops stay that way or any of the why's about it. The main thing is that it's pretty or it's pleasing and it's really an effort to care why." I remember Jean and Sandy nodding in agreement.

I cared very much about the why's and I was still puzzling about the paradox of soap weakening water drops but strengthening bubbles. David had called soap bubbles more stable and had said that using the word strong got us into trouble. I didn't understand that. Since the word stable did not mean much to me, I wanted to know why David objected to my calling the soap film strong:

David: You noticed that this bubble lasted longer?

Maja: Yes. That's because it is stronger. Now I can't say that anymore.

David: Yes, but you have a because in there which I wouldn't have. I was just saying it lasts longer. I'm just describing a thing that means it's more stable.

Maja: I see, lasting longer means greater stability to you. It means that?

David: It's just the same thing. It's not an explanation of it.

Sally: He's defining stability.

David: It's not an explanation of anything, it's just a description..

Maja: It always means lasting longer?

David: Yes, it's always protection against shocks or disturbances. You say, "all that I observed about the film is that it lasts longer. I don't see that it's physically stronger in any other sense." Then you can go back and say, "Well, it could be possible that it was physically weaker, that is, easier to stretch, and yet last longer." There is no sense of contradiction there, whereas when you use the word stronger you seem to be inviting a contradiction.

I finally accepted the word stable as describing the soap film, but I was still not satisfied. It was a description and I wanted an explanation. I had this funny feeling that stability carried an additional meaning to scientists. I wrote in my notes: "Stable: what does David know about it that I don't know?" Was there another meaning that I was missing or did David simply not want to go any further with the soap bubbles? This past week, when I was going over the transcripts and still found myself confused and dissatisfied, I again asked David for an explanation.

This time he brought in molecules. Soap molecules, he said, are much more complicated than water molecules. Their two ends react differently to water. One end is hydrophilic, the other is hydrophobic. In a soap bubble, these molecules line themselves up in such a way that they act as stabilizers of the thin soap film. They help to keep the film at an even thickness while it is being stretched and this keeps it from breaking.

I realize that I could have continued to ask: what exactly do the soap molecules do and why are they doing that? But just as Hedy was satisfied with the description of the water drop and the soap bubble, I am satisfied right now with the description of soap molecules and I can accept the fact that their behavior accounts for the stability of the soap film without wanting to know any more.

Molecules have now become part of my thinking although they are still very much on the periphery of my mental framework. Because it is easy to talk about molecules and atoms, to use the labels without understanding the concepts, David wants students of all ages to have a broad knowledge built on observation and experimentation before talking about the physical world in terms of atoms and molecules. When we were studying heat and asking a lot of big questions about electromagnetic radiation, David told us that he would like to have kept us in the 18th and 19th centuries for a while so we could arrive at an understanding of heat which paralleled the historical scientific developments of that subject. We were too impatient; we wanted 20th century answers even though we find them very hard to understand.

How do you know when students are ready to think in terms of atoms and molecules? Timing is always one of the most difficult questions which teachers have to face. David, I think, prefers to err in the direction of being too late. Most science teaching errs grossly in the opposite direction.

Cubes, Bananas and Plasticene

Since neither the soap froth nor the water drops raised the question of area/volume relationship in teachers' minds, David and Ron decided that the topic might be made more accessible if we spent some time working with wooden cubes, where the changing relationship could more easily be analyzed. This is how David introduced the cubes:

David: How many faces does one of these little things have? (They were 3/4" cubes.)

Teachers: Six.

David: OK. Now, if you make the next bigger cube out of these little cubes, how do you do it? (Teachers made a cube that is two little cubes long, wide, and high.) OK, there is the next bigger cube. How many cubes in that?

Teachers: Eight... four... oh, right, eight!

David: The first one was one cube with six faces. Let's give these faces a name, (Group discusses possible names and settles on "minch.") There are eight little cubes in this next bigger cube. It's 2 x 2 x 2. Now how many minches are there on the outer surfaces?

Teacher: Twenty-four.

Teacher: How do you get that?

David: You multiply, because you observe that there are six sides. It's still a cube and each side has four minches. OK? Then, what's the next bigger cube? How many little cubes in the next bigger cube?

Although somebody correctly answered "27", I asked David to slow down so teachers could take their time building this "next bigger cube." I sensed that several teachers were already quite confused.

Having been rather open-ended in the previous class, David was quite explicit in his instructions about how to make the cube grow, hoping that the change in area/volume relationship would now become apparent. Little did he know what kind of trouble we would get into later because he told us to make "the next bigger cube."

Teachers reacted in many ways to this class. Hedy didn't like it. She hated the math. She reports having one fleeting insight "oh, that's what it means to cube something, literally make a cube of it" but when she told someone about it she realized that

> "as soon as I said it I already couldn't remember or understand what I had just said... I went on jotting down the numbers of cubes that would be in each succeeding size of cube but I was just multiplying. It didn't get any realer. I also tried figuring some ratios because I overheard Ron suggesting this to another person, but the ratios didn't mean much. So what? my usual feeling. Mostly it was an enormous strain to try to think about it so hard, so profitlessly. I went home with a headache and I never have headaches!"

Sandy, on the other hand, reacted in the opposite way.

> "I was completely delighted with that session...The mathematics of surface and volumes was wonderful. I felt the full power of discovery as the relationships began to unfold before my eyes.

I love number patterns too, and I was going to make a chart which would show every possible measurement of cubes. I had columns for length, n; area, n^2; surface area, $6n^2$; volume, n^3; and number of inside faces, $6n^2(n-1)$. David suggested that I add the inside to the outside faces and showed me how that becomes $6n^3$. I spent the whole class time writing down growth patterns in these different columns and realized once again that there is no such thing in math as a chart that would show everything. As you begin to see patterns and relationships, new ones keep appearing that can be added to your chart. I got so engrossed with all my number patterns that I completely forgot about the purpose of the class -- to explore area/volume relationships!

Sally had a similar experience:

> "Working with cubes was fun, but I became so absorbed with trying to find formulas that I totally digressed from what I was first interested in finding -- the ratio of surface area to volume. In fact I wasn't thinking of the science work at all, I was just enjoying myself."

It was Ron who tried to bring us back on course and suggested that we try to find the ratio between the volume and the surface area growth of the cubes. What happens when the linear dimension of an object is doubled ("the next bigger cube")? We learned that the surface area of that object quadruples and the volume becomes eight times as big. (Area increases by the square of the length, volume by its cube.) For some teachers this was an exciting discovery, for others a source of great confusion.

Myhra reported that after she got home,

> "thinking I really knew what I was doing, I found out the next day that I had the formulas for figuring out area and volume reversed. It all seemed to make sense to me at the time. Now I'm not sure I really do understand what appeared so simple earlier."

When Ann wrote her weekly notes, she found herself

> "trying to make sense out of the relationships of volume to area. I feel I need to get out the cubes again. As the volume gets larger, the area gets smaller because there is more space for the faces of the cubes to be hidden in the interior of the cube. Is that right?"

Sandy, on the other hand, found these relationships "crystal clear" but added: "What is not so clear is what this has to do with nature." Shelley also caught on to the relationship between area and volume, but like Sandy, she went home "still looking for what all of this meant in 'real life.'"

When David decided to use the wooden cubes, he was not planning to make an analogy between their inside faces and the interior soap film which increases so rapidly as you beat the soapy water. But somehow the mention of inside faces during the class made some people think that there was meant to be a connection. Mary Jane asked: "If there is any correlation between surfaces of soap bubbles fortifying themselves when they come together, are those molecular fusions of a different quality than wooden cubes which don't adhere to one another?" I had made the same observation: the more little cubes you used to build bigger and bigger cubes, the more wobbly the whole thing became. "It does not get stronger like the soap film," I said to David, "it falls apart." Somehow, I thought that the inside cube faces were supposed to illustrate why the soap froth got stronger as you keep subdividing the interior film by beating it. The inside cube faces did come up for discussion in class but not for that reason. It was found that you could set up another ratio with their growth in relation to either volume or surface area growth. (I didn't realize that until today.)

The next day, at our weekly staff discussion, Ron expressed surprise that teachers who understood how a cube grew had a hard time figuring out how volume and area changed if the process was reversed and the cube was made to shrink. That didn't surprise me in the least. I could see why Ron might think that if you could do the calculation in one direction, the reverse would be obvious, but to me -- and to most of the teachers -- it seemed like an entirely new problem. In fact, I got quite confused when David tried to help me understand this: "Pretend you have a nice big cube," he said, "and you saw it right there in the middle, this way. Now you don't have a cube anymore. Now put the pieces back together again in a vice and saw them in the other direction...:

Maja: Then you have four cubes.

David: No, no, no, you have four pieces that are twice as long in one direction.

Maja: You mean with two cuts you don't get four cubes?

David: No. (David describes how you have to cut the cube to get back to a smaller cube.)

Maja: So you have to have three cuts?

David: Yes, three cuts. How many cubes will you get out of this?

Maja: I suppose eight.

David: You suppose? What do you mean you suppose?

Maja: I'm saying that because I know the number from working with my growth patterns but if I didn't know that and had to think of the actual cubes, I'm not sure I could figure it out.

Shrinking a cube seemed much more difficult than making it grow, probably because the three-dimensionality of the cube wasn't quite real to me. I made that discovery at the same meeting, the day after the cube class.

Ron related how a teacher's "eyes lit up when she discovered that the little 2 above a number actually meant a square and the little 3 referred to a real cube." I remember my own pleasure some years back when I realized what squaring a number meant, but I don't think I ever visualized a cube when people talked about cubing a number. The "little 2" had become related to a geometric square; the "little 3" had remained an abstraction. I now became intrigued by these geometric representations of powers and wondered whether perhaps there were other shapes, such as a tetrahedra, which represented powers past three. David said that my analogy was right but that in the physical world you can't go past three without artificial inventions. You can go to higher powers but you don't have any geometrical representations for it because space is three dimensional. In the real world there are only three directions: east/west, north/south, and up/down.

When David said this, something clicked in my mind. The cube suddenly became a real-world three-dimensional object which therefore had to grow in three directions. If you increased the length of the cube from 1 to 2, you also had to increase its width and height from 1 to 2. That's how you got 2 x 2 x 2 or 2^3. I had known the formula for many years but I had not grasped the logic of it. Now I can say, obviously the volume of a cube (or of any object, as I was to learn later in the course), growing in three-dimensional space increases faster than the two-demensional area or the one-dimensional length. Why had I never realized this before? Even though I had played with squares and cubes of different sizes to get the visual picture of these growth rates, the real meaning of these relationships had escaped me. Now I have crossed this barrier and have arrived at a new plateau in my understanding. What I still miss however is fluency with this idea, a fluency which comes only from experience in everyday life and from much thinking about size and scale in the physical world.

I had another strange insight that day which I recorded in my notes: "If there is this relationship between linear area, and volume growth, then everything that has size must be affected by it. Since everything in the world has size, does that mean then that everything in the world is governed by this law?" No wonder David keeps coming back to this topic as providing us with a powerful organizing idea .

David commented on the many confusions that had arisen in the cube class:

> "I have this hypothesis that the notions of large and small all have these conventions in our mind long before we learn anything about numbers. It's a mixture of all these things we are talking about: length, area, and volume, a mixture of things that are not distinguished from each other by common sense. They are just taken intuitively as big and small, so if something is twice as big in one sense, it can't be four times as big in another sense

> because there is only one sense of big that you recognize consciously... the common sense idea is an undiscriminated mixture of two or three ideas which the scientist or the mathematician wants us to sort out and use independently of each other. And we can't do that until we become reflective about it and see the need in terms of some interest of our own, some curiosity of our own. Does that make sense?"

It made a lot of sense to me. I puzzled about this problem in a slightly different way many years earlier when I was first introduced to the topic of size and scale. I was playing around with squaring and cubing different numbers to get a feeling for area and volume. I suddenly became concerned that the same number symbols were used to describe very different kinds of measurements. I had written in my notes:

> 2^2 equals 4. 2^3 equals 8. Four refers to an area enclosed by four lines of two units each, but what does the four really stand for? How can a length and an area, which are so different, be described by the same symbol? The cubic measurements are even more difficult to comprehend. The fact that the "4" is followed by "square foot" and the "8" by "cubic foot" doesn't seem to make enough of a difference. These are just words without associations or meaning.

The cube class brought me back to my confusion about the meaning of the words area and surface area. In the past, when David had talked about the surface area of leaves on a tree, it had never occured to me that in his mind, the surface, which to me implied only the top boundary -- like the surface of a pond -- would include the entire outer layer of an object. For leaves, it would include the top and bottom, on a table, the top surface, the underneath of that top, the edges, as well as the table legs. In my mind, area had always been flat, facing in just one direction. When I heard the term surface area, and couldn't visualize it as flat (as with the root hairs mentioned in our first meeting), I became confused. The surface area of the cube included all six of its faces (as well as imaginary interfaces if you chose to include them) and that was difficult for me to incorporate into my notion of area.

This notion was a mixture of high school geometry -- rectangles, triangles, circles, always drawn on a flat piece of paper -- and area as it has come up in my everyday life -- a floor to be carpeted, a wall to be painted, a lawn to be fertilized. Even in real life, area was always flat. (When I use my common sense, however, and don't think about what area is supposed to mean, I know that if I want to cover the area of a table with paint, I have to buy enough paint to cover the entire outer surface and not just the table top!)

Because my confusions are often indicators of similar confusions among the teachers, we planned to devote the next class to further explorations of area. "In the physical world." David said, "area is always the area of a real surface." We were going to get real-world objects with easily removable surfaces -- fruits and vegetables which could be peeled. Then, David said, we could transform the curvey outside surface of irregularly shaped fruits and vegetables into something that is represented on a flat piece of paper, as the area of geometry texts.

In class, before working with our edible materials, David asked the group: "How do you think about the area of that table over there?"

Ann: "I think of just the top of it. When you talk about surface area, it would be all the exterior you see."

Maja: Before this class, what was your image of area?

Myhra: Maybe a rug, an area rug.

David: And how do you specify the size of such a rug?

Teachers: In square feet...by the exterior beneath it...8 x 10...

Myhra: You're buying a rug to fill a space and you give the dimensions, like 8 x 10, so when they come to sell the merchandise you know whether it will fit or not. As opposed to, if you knew how many square feet you had in your living room and you go to look for that number of square feet in your rug.

David: It might not fit.

Sally: It might be the right number of square feet but a different shape.

Myhra: Yes, you want to know how much contact the rug is going to make with all the surface.

Shelley: Does the rug have two surface areas? Do you count the top and the bottom?

David: You see how really complicated this turns out to be?

Sally: And when you're talking about acreage outside in the field, that's different too. It's not just flat, it's up and down.

David: That's right, but you sort of treat it as flat, don't you?

By raising questions about area, by accepting our way of thinking, and by admitting that he hadn't really thought much about the difference between area and surface area -- "I've never been conscious of either using the word surface or not using it, so it's very useful to me to realize that that caused trouble" -- David got us to think much more deeply about the meaning of area. We wondered whether both sides of a surface should be counted and how a hilly piece of land was measured, and then Ron challenged us even further by asking what the area of a square foot of velvet would be -- "is it a square foot or the area of all these little hairs?" As I am writing this, I began to wonder what is included in the measurements of large areas of land, like national forests? Do they measure only the surface of the land itself, or do they include everything that grows on it or protrudes, like trees, rocks and mountains? I do know that surveyors wouldn't include the surface area of all the needles in a forest of evergreens but I'm having fun thinking in this new way.

At our staff meeting, we had wondered whether to put out graph paper for the work with surface areas. David would have preferred teachers to approach these explorations by comparing different surface areas without immediately going into numerical measurements. "I'm a little bit nervous about prematurely dividing something up into squares," he said. "I think there is some intuition of quantity that doesn't have to be translated right away into numbers.

The fruits and vegetables were invitingly arranged on a table when the teachers arrived for class. "These are friendly forms," wrote Hedy about her initial reaction when walking into the room. "No matter what we do with them, I will like this." Everyone seemed to feel the same. As soon as the teachers had peeled the fruit, they asked for graph paper. Initially we tried to withhold it but Sandy was insistent; "how can you find out the surface area of a banana skin without graph paper? You have to have a unit to give an area measurement." So I handed it out. We probably didn't sufficiently stress the point about comparing surfaces. When Mary Jane wondered whether she could find the surface area of a zucchini by wrapping a string around it, we should have encouraged her to try it and then try to wrap other fruits, or different size zucchinis, to get comparative amounts of string. At the end of the class, David mentioned that the area occupied by a peeled orange could be covered with rice grains and when this was done with areas of other fruits, the quantities of grains could then be compared. Perhaps we should have mentioned such possibilities to the teachers to keep them away from graph paper.

Most teachers peeled different fruits and then laid the peels out on graph paper to determine area. They enjoyed the work, but again several people wondered why they were doing this. Mary wrote that she was impressed to learn "that seemingly compact shapes could have a significant surface area," but she wondered why one would ever want to know how much surface area an apple had; "isn't there an easier way to puzzle it out than peeling and laying it out, or peeling and weighing it? What value is this knowledge to me?"

We had also set out balances so teachers could get at surface measurements by weighing sections of skin and setting up ratios. The weighing was a big success but the idea of setting up ratios caused considerable trouble. Myhra wrote: "Why did I have to weigh a square inch? Couldn't I have weighed a quarter of that? Is the square inch part of the formula? Does it make any difference to use inches and grams in the weighing method?" I know from my own experience that ratios present a big barrier. I didn't really understand fully until now, going over all the transcripts and notes in preparation for writing this essay, that a ratio is a relationship of numbers and that the numbers can be quite different but still have the same ratio. I must have understood this on some level but whenever the word ratio is mentioned, I seem to have to stop and think what it means. It is not easily accessible knowledge.

I suggested to Shelley that we try weighing the area of her palm. She drew around her hand and cut out the imprint on a piece of cardboard. She then made some units out of the same cardboard -- squares which had areas of 1, 4, 9, and 16 square inches respectively. She weighed the cut-out of her hand on an equal arm balance and found that it balances with the 2" and 3" squares. Now what was the area of her hand? First we thought it was 5 square inches, but we quickly realized that was wrong. Then we figured that we had a square of 9 square inches, and one of 4 square inches. Could we add those together? We thought the answer might be 13 but we really weren't sure if square inches could be added.

When I told David about our problem he said: "You could have cut the hand up into two pieces and you would have known that you could add them to make the hand again; you could have cut up your squares into nine and four smaller squares and then mixed all these squares up together and you would have known that you had thirteen." How obvious, I thought. Why were we so confused? Here was a good example of using numbers without having a real understanding of what they mean.

Then David explained:

> Add doesn't just mean arithmetic add, it means physically put together. You can arithmetic add because you can do the other. When a child adds two handfuls of pebbles together, that is adding. We get the arithmetical meaning of the word from that, but it wouldn't mean anything if somewhere in the background we didn't physically put things together... amounts of surface can be added and divided. You think of division not the way you think of arithmetic but the way you think of scissors and you think of adding as moving two piece together or rearranging them...You can add areas arithmetically but you can also just put them together and see that they make an area twice as big. The meaning of add and subtract and divide that a child knows with the physical operations and not with numbers is the meaning you need to recover here.

We have moved so far away from these original meanings that we couldn't solve the simple problem of adding 9 and 4 square inches!

If we had a feeling for area, the little puzzle of whether four square miles is the same as four miles square would also be easy to answer. Instead, many of us probably felt like Hedy who, after having the difference explained, exclaimed: "That's incredible! I had no conception that that would be different. The words don't give you a clue. Having square and mile and a number in the same sentence -- they could be in any order and they would all sound the same."

Most people worked only with the concept of area in this class, but some teachers got into volume -- another big stumbling block. Sandy reported: "I understood when I worked with the cubes how to get from length to the total surface area and the total volume. Then I thought that there must be a way to just simply measure the banana and from that length measurement get a total surface area and total volume, using that formula. I was wishing it would be so easy but with the banana, there is a strange number there and it's not clear and I don't know what it is or how to find it."

Sandy's problem brought us closer to the question of whether the relationship between volume and surface was peculiar to the cubes or whether it held for all shapes. "That really is the big step," said David, "and that is the step that almost never is taken anywhere in school." David then talked about exploring surface/volume relationships in a different way: Could we change the shapes of things and have the area stay the same? What would happen to the volume? Would it stay the same or would it change although the area remains constant? Could we compare a whole lot of things of different shape that have the same amount of surface? What will be the difference in their volume? Which shapes have the most and which shapes have the least volume? These were new questions which most of us hadn't even thought of.

At our next staff meeting we decided to get a lot of Plasticine so teachers could make differently-shaped objects and then change their shapes to study area/volume relationships one more time. We also thought Plasticine would be useful for those teachers who still had difficulties understanding what happens when you make a cube smaller. It would be easier to cut up a Plasticine cube than one made out of wood.

Like every other class so far, the Plasticine class revealed additional confusions. A number of people worked with Ron, changing the shape of a piece of Plasticine from cube to sphere to pancake to snake, observing how the surface area changed while the volume remained the same. Others got interested in a question that Marsha asked: "Suppose I have an apple, and I want twice as much apple?" Any child would probably know how to get "twice as much apple" but we suddenly became confused! In the examples given us in class, the linear dimensions were doubled, so the volume was always eight times larger. We now wondered what you had to do to get just double the volume? Marsha decided to work on this question and started to make a Plasticine apple. Several teachers decided to make Plasticine cubes and then try to double their volumes. First they made two cubes of the same size, then they squashed these two cubes together to make one larger cube. Although we all knew this new cube to be twice the volume of the original cube, we didn't think it looked twice as big. Myhra suggested we make three cubes of the same size so that after the double volume cube was made, we could compare it with the original cube. Everyone was surprised and somewhat disbelieving that the double volume cube looked so small. I believe all thinking about how cubes grow went back to the class in which David introduced the topic by telling us to make "the next bigger cube" out of the 3/4" wooden cubes, where the next size meant doubling the length and therefore getting eight times the volume. We completely forgot that in real life there could be an infinite number of in-between cubes. The volume of cubes grows dramatically when you double the linear dimension, but obviously cubes can grow at any rate you choose.

The relatively small "double volume" cube led to a whole discussion of size; what does "doubling" something really mean? Do you always have to specify what dimensions you are doubling? Do people who are familiar with this concept just automatically think of the area as being four times as large as the length and the volume as being eight times as large when the linear dimension is doubled? We realized that we had never made these distinctions when talking about size, and we also realized that our intuition of volume was rather undeveloped.

A few teachers wanted to see what would happen when you cut a cube in half. They made a large Plasticine cube, the same size as a 3-unit wooden cube (27 little cubes), and then cut it in half - one cut in each of the three directions. They expected the resulting eight smaller cubes to be the size of the wooden 2-unit (8-piece) cube. Why, they asked, were their Plasticine cubes smaller? Common sense would tell you that half of three is one and a half and not two, but since we had always made the wooden cubes grow by units of one, a 1½ unit cube didn't fit the model.

David later said that it was probably a mistake to have used the wooden cubes to introduce the idea of three-dimensional growth. "We have been going in multiples of that unit, and that is inessential to the idea." It may have been inessential to the idea, but it is where most of us got stuck in our thinking because we didn't yet fully understand the idea, nor did we know where it was supposed to be leading us. Hedi wrote: "Part of my lack of interest in these size topics is that they are so abstract. Cubes are really meaningless to me. They explain something to somebody but I haven't yet asked the question that they are supposed to answer."

Another rather extraordinary confusion arose in this class. Sally wanted to show that cubes of any size would grow in the same proportion as the 3/4" cube we had been using. She took a set of Cuisenaire cubes, where the smaller unit was 1 cubic centimeter, and stacked them up to show how their volume changed each time the length was increased by one unit. Several teachers (including myself) were surprised that the size of the Cuisenaire cubes didn't increase as rapidly as the 3/4" cubes. It took us a while to figure out that the cubes grew in proportion to the original unit, and that this proportion remained the same for all cubes, although their actual sizes can be different. This insight led to another interesting discussion about the meaning of one. If the number one cube can be of any size, then what does one really mean? Mary wanted to know. How can one describe a cubic centimeter and a cubic inch cube?

I often wonder why we fail to use our common sense when we are learning something completely new. Our confusions over size reminded me of the problems of traditional teachers who sometimes have a difficult period of transition when they want to change their teaching approach. They tend to give up all their useful traditonal teaching skills as soon as they start trying new approaches as if the old ways and the new ways couldn't be combined. Similarly, when we are confronted with new science concepts which we don't yet fully understand, we don't seem to use any of our common sense, perhaps because we have to restructure much of our earlier thinking, even though much of it could still help us with questions and confusions.

It makes good sense that after our exposure to length, area, and volume growth, we would start to wonder what big really meant. Our older, generalized notion of size had to be refined so that we would differentiate between longer or heavier or having more area or more bulk. On the other hand, it doesn't make any sense that we should think about growth in terms of "the next bigger cube" or that we would expect half of three to be two or that we should think a cube made of 27 one ccm pieces would be the same size as one made up of 27 3/4" cubic inch pieces. We certainly wouldn't expect a mouse or a dog or a horse to be the same size if we were told that they all doubled their volume or weight in the first three months after birth!

It seems that when a subject is new and unfamiliar and we feel insecure and have little knowledge or experience to draw on, the example with which we learn something often becomes identified with the idea itself. Most of us got confused all over again when David and Ron switched from saying "when you double the length, the area is four times as big and the volume eight times as big," and talked instead about area being the length squared and volume being the length cubed. Because we learned this rule with the specific example of doubling, we took the example to be the rule. Myhra wrote in her notes, "I'm trying to remember now that the 3rd power is cubing and volume is always cubed. Can I say: when you cube something, it is eight times as much? I think so."

I don't remember the Plasticine class as being frustrating. We laughed a lot about our troubles and we thought a lot about what we were learning. Shelley, though, must have hit a real stumbling block when she started to explore volume with the Plasticine. She wrote: "People went in different directions. Some seemed to know with confidence what they were doing and others (like me) watched, looking for a place to start. I felt the same despairing frustration I always feel when people start to apply formulas from their memory and say: ' just do this...it's simple, really.' We're all at such different levels of experience."

The Elephant

Several times at our staff discussions, David had talked about some of the applications of size and scale in the biological world. Sally and I were fascinated when he described how the digestive system and the breathing apparatus change from simple one cell creatures to animals the size of an elephant. He talked to us about "the fundamental biological fact that living things have to maintain roughly the same area of exposure to their air and food supply as the small things do that get it through their outer surface... The ratio of surface to volume or surface to mass remains constant." He then explained to us how food diffuses through the surface skin of an organism "so that the amount that can get through is limited by the amount of surface area, whereas the tissue to be fed is three-dimensional. It's a very basic fact that the architecture of living things is accomodated to this and that's why little things are different in shape from big things."

Sally mentioned that several teachers had been asking questions about Julian Huxley's essay The Size of Living Things which we had been given at a previous class and that it might be a good idea to leave our mathematical struggles for a while and devote a class to talking about the biological implications of size and scale. It was decided that Ron would speak on this subject at our next class meeting.

Ron chose the example of an elephant to start his talk. He assumed that by now we understood what happened mathematically when you scaled an object up or down, keeping its exact shape, and he wanted to show what would happen to an elephant if his linear dimensions were doubled. As he started, however, teachers interrupted him with questions which showed that there were still plenty of confusions. I would like to quote the beginning of this class:

Ron: Let's take an elephant. He weighs 5 tons, 10,000 pounds. His main body part is about 10 feet long. So let's make the elephant twice as long and that means twice as long, twice as thick and twice as high. I'm going to double his linear dimensions.

Maja: From where are you measuring him?

Ron: I'm going to take his square body. (Ron had drawn a sort of cubist elephant on the blackboard.) What happens now if I make my elephant 20 feet long, 20 feet wide and 20 feet high? I've doubled all the linear dimensions. I've made a gigantic model and everything is twice as long. How much bigger will the volume be?

Mary: I don't know how to figure out the volume. I know we've figured that out but at this moment I don't know.

Sally: Those things really leave you fast, don't they?

Ron: (Uses the wooden cubes to demonstrate the volume growth.) Twice as long, twice as wide, and twice as high. Eight times the volume.

Mary: (Building her own model) 2 x 2 x 2.

Ron: Here is my first elephant. If I make him twice as long and keep him the same shape, I have to make him twice as wide and twice as high. So he's eight times as big in terms of volume and therefore weight. There are eight of these elephants (pointing to the cubes) in that.

Mary: Are these arbitrary numbers, the 10,000 pounds?

Ron: That's about right for an elephant.

Mary: But what does that have to do with the 10-foot dimension?

Ron: An elephant is about 10 feet long and weighs about 5 tons.

Mary: OK, but you could have used any number, and then would the second number relate to it?

(Mary is asking an important question but at this moment, Ron could not take the time to deal with it.)

Ron: That much elephant is 10 feet long and it weighs 5 tons. I don't think that we ought to get too tied up with number relationships. OK, if I make him twice as long, to keep him still looking like an elephant, I'm going to have to make him twice as wide and twice as high. I'm going to have eight times as much elephant so I'm going to have 40 tons of elephant.

Jean: If you make him twice as big, do you double him too?

Ron: Look, I did this, I doubled all the dimensions so I've got eight times as much elephant. Does everyone see this?

Mary: Yes, I do understand that. I guess what I don't understand is the connection, well, so you say he's ten feet across, ten feet high and ten feet wide. So you multiply the ten times eight to get some number which is eighty.

Ron: All I'm saying is: my original cube weighs 5 tons and I'm going to have eight times as much. There are eight of those cubes in my new elephant so he's going to weigh eight times as much.

Jean: If you wanted twice as much elephant, you wouldn't just go get another 5 ton elephant?

Ron: I wouldn't know what to do.

Sally: Well, now if you said you wanted twice as much elephant, maybe you need an elephant that weighs twice as much.

Ron: Yes, I think you might.

Jean: That's what I'm trying to find out, that's what I said about doubling and twice as much being the same thing.

Ron: But this is a new kind of creature we are inventing now.

Mary: Even if you doubled the linear dimension, you could double the weight and have twice as much elephant.

Maja: Or even the area if you chose to double the area.

Ron: Yes, but I'm being very precise and I didn't say I was going to have twice as much elephant, I said I was going to double each linear dimension. There are three: twice as long, twice as wide, twice as high. 2 x 2 x 2 = 8.

Jean: I'm having trouble with what you call that 8, is that the volume? And what is the 5 ton?

Ron: That's the weight of the original elephant.

Jean: What's the difference between weight and volume?

Ron: It just happens that this much elephant weighs 5 tons. That much wood would weigh 3 tons, or that much lead would weigh 100 tons. But a piece of elephant 10 x 10 x 10 just happens to weigh 5 tons.

Ann: I'm getting confused between twice and doubling.

Myhra: Doubling and twice as much? They are not the same, right?

Ron: In linear dimensions, if I make something twice as long, I'm doubling the length, aren't I? And I was very careful to say: we will double the linear dimension.

Ann: OK, so are you saying they are the same?

Ron: Twice as long, double the linear dimension.

Sally: I want to go back to what Jean was talking about, because I don't think the 5 tons was a relevant thing. The elephant could have weighed 4 tons, he could have weighed 5 tons, or he could have been a baby elephant and weighed 1 ton. But whatever he weighed, if you double his length, he is going to weigh eight times as much. So if he was a baby elephant and weighed 2 tons and you doubled his length, width and height, he is going to weigh 8 x 2 which is 16. He is going to weigh 8 times as much because he's got eight of those (cubes).

Jean: It's even more basic than that, I don't understand what the definition of volume is, I guess.

Mary: I think that is what I was getting mixed up with too, wondering if that 10,000 pounds is somehow a result of his dimensions.

Reading over this transcript, I really sympathize with Ron who tried his best to answer all our questions though I don't think he always knew what we were asking. After Jean's admission that she didn't understand the definition of volume, there were many more questions asked about volume, weight and mass before Ron could return to his elephant:

Ron: I now have a 40-ton elephant. Now let's look at his legs...

Myhra: May I say something before we get to the legs? In the teacher's textbook for kids, they have this story problem of the elephant before the kids have the basic concepts to be able to look at the story problem; and therefore they blow it and they don't like story problems and they feel very frustrated. That has happened right here. The cube thing kind of made it all clear to us but you start on the elephants and we were just all over the place with questions and then we got back to the cubes and finally zeroed in on what you wanted to tell us.

Ron: Is this OK now?

Mary: Yeah, on to the legs.

Finally Ron was able to make his point: if the length of the elephant was doubled, the cross section area of each of the elephant's feet would quadruple but his weight would be eight times as much. Each square foot of leg would therefore have to support twice the weight and that wouldn't work. "The weight has gone up 2 x 2 x 2 and the area supporting this weight has only gone up 2 x 2." I think the idea of the cross-section area of the elephant's foot was more than Ann could handle. "How do real elephants grow?" she asked.

Time and space don't allow me to report all the details of the continuing confusions of this class. (For complete transcript, see Vol. III, pp. 119-140.) In the end Ron did get to talk about some interesting biological facts, for example, that the surface area of the human lungs is supposed to be about the size of a tennis court. The class ended with Sally wondering how that could be possible: "I can't imagine folding a tennis court over and over and over until it would fit inside me."

At our staff meeting the following day, we discussed the teachers' questions about volume. As usual, I shared many of their confusions.

First, I found out that I rarely ever thought about the volume of an object. If I wanted to describe how a cube grows, I would just say that it got bigger "all around." I wouldn't say that its volume had increased. I realized that I didn't even think of volume as a unit of measurement! I was also troubled by the fact that volume could refer to a solid object and at the same time to an empty space. For some reason I resisted thinking of volume in this double way. In class I had asked Ron: "How are you supposed to think about volume? Is there one way I should think about it that is more correct than another? How do most people think about volume -- as something empty or hollow or as a hunk of something." David tried to explain the two concepts to me: volume as capacity -- "a container that defines or surrounds a certain piece of space, a set of walls within which you can trap something" -- and the other idea which he called bulk -- something that takes up a certain amount of room. "there are two ideas of volume that have to be connected; one is a container that has a certain capacity and the other is what fills the container." David wondered why we had trouble connecting these two ideas in our minds, since they were so closely related. Right now I am wondering the same thing: having made the connection and assimilated the double way of looking at volume, I cannot remember what troubled me last year.

I also had a hard time thinking of something spread out or thin, like paint on a wall, as having volume. I know that paint has volume when it is in a gallon can. But once the paint is on the wall, it lost its three-dimensionality for me and seemed to become part of the area of the wall.

To help clarify some of these confusions we planned to assemble a large variety of containers of different sizes and shapes for the next class, as well as salt, rice and beans, and paper to make cones and cylinders.

Sally then reminded David that at a previous staff discussion we had talked about the fact that all shapes, not just the cubes we had worked with, follow the same growth laws. This came up when David tried to help me understand why there couldn't be a mountain 100 miles high -- "the base would have to support more and more weight for every unit of surface and if the mountain gets too high the rock will bend and the earth will begin to act like a liquid... the crust of the earth will be soft from these enormous pressures."

Sally and I had been very excited by this new information which we were sure the teachers did not know. Sandy had been asking her in her notes: "What happens to the growth when an object is irregularly shaped? Natural objects are not cubic but asymmetric and uneven and I don't see how one can even take a linear measurement on most real world things." Sandy's questions, as well as the confusions of the elephant class, made it clear that this idea had not been adequately dealt with. We hoped that when the teachers worked with the materials we were preparing they would be able to extend their understanding from cubes to other shapes.

David believes there is a time in people's learning when theory can help pull things together. Before our work with volume, he wanted to talk one more time about the relationships of length, area, and volume. He hoped that we would be able to go beyond numbers and become comfortable with the knowledge that if you change any one of these measures, the others will change in a constant ratio.

In class, he started out by saying that he wanted to get away from the cubes, which only allowed us to increase things by fixed amounts.

"...we really need to get away from fixed units that you count. Think instead in terms of length and areas and volumes as quantities that can change by arbitrary amounts."

David then defined length (or the linear dimension) as the distance between any two fixed points on a shape -- "if it's the elephant, it could be the distance between the tip of his tusk and his tail or the distance between the bottom of his feet and the top of his back... you can pick anything as long as you pick the same length on the small elephant and on the big elephant." Area, David continued, could be the bottom of the elephant's foot or the surface area of his trunk or "all of his skin area all the way around." Sally thought it was neat that you didn't have to think about his entire surface area but just one part of it. "If you preserve the absolute similarity as you scale the elephant up or down, it doesn't matter which, you can get away from any worry about do you count the area on the bottom of his feet. You can but you don't have to, it is your choice," said David. He did not say much about the third quantity -- volume -- except that we would spend some time looking at it in class. And then he gave us "three simple sentences":

1. The area is proportional to the square of the linear dimension.
2. The volume is proportional to the cube of the linear dimension.
3. The ratio of the volume measure to the surface measure is proportional to the linear measure. (David said that this statement was "the big one, the clincher" but we shouldn't feel that we had to understand it right away.)

The main thing to remember was that if you scale an object up or down, if you keep it the same shape but make it larger or smaller, "the length or distance and the area and the volume all increase or decrease together, but not in the same proportion... there is a fixed relationship between these three quantities so long as you keep the shape absolutely the same."

There were many questions during David's talk and we didn't have a great deal of time to work with the volume materials which we had prepared. **When** we started, however, it quickly became clear that our intuition about volume was quite poor. I found it hard to believe that a cube ten centimeters on each side would hold as much liquid as a one liter bottle. Marsha related how she had ordered an end loader of dirt: "I thought I wanted two until I transferred the one load with a wheel barrow down to my garden. I couldn't believe how much it was." And Sally told how she ordered ten tons of gravel -- "I figured out mathematically how many square feet I was going to cover by what they told me, and when they dumped it out I said: 'that little pile?' I thought that I had gotten gypped until I started to spread it out."

We have little experience with volume in everyday life, except with familiar capacity measures like cups, pints, or quarts. I wonder therefore whether we might be judging the "amount of stuff" by some linear dimension. Maybe we are just looking at the length or the height of a pile of sutff, or even at the length and height, but somehow, as Sally said, "we are not visualizing those three dimensions." We obviously needed to devote more time to volume.

Toward the end of the class, I asked David if he could spend a few minutes talking about where our new understanding of scaling would take us if we gained more fluency with it. He said:

> For Ron, it would be the fact that you cannot scale living things while keeping their form exactly the same. The only way you can succeed in getting bigger living things is to change their form. It's the nonscaling involved in living things that's a fascinating and very rich topic. For me, in a much wider sphere of application, it has to do with the way the properties of things change with size in general, not just for living things. Why little drops of water would sit on a wax paper and make all those perfect spheres while big globs of water will flatten out and make almost flat surfaces. Why things that are bigger than the planet Jupiter are fiery hot and why things smaller than the planet Jupiter are apt to be fairly cold. Why things bigger than the moon are always round. If these things were well learned and acceptable in the imagination, they would give you a kind of classification system for all the furniture that has been discovered to exist in the natural world from atoms to galixies.

Sally wanted to know why things that are bigger than Jupiter were fiery hot.

> Things that are bigger than Jupiter get squeezed so much by their gravitational pull that the atoms that can maintain themselves in the cool state get crushed and that leads to nuclear reactions. Hot, hot, hot stuff. What's the biggest thing that can be shaped like this tomato can? (one of the containers we used in our volume work) If it gets bigger and bigger and bigger, that gravitational pull is going to dominate finally and it's going to squeeze it together and make it more like a sphere. And that goes back to Ron's example of how smooth the earth is. Mt. Everest's height is only a tiny, tiny little fraction of the four thousand miles which is the earth's radius...
>
> Why is this important? It's a very unifying thing. It's not detailed, it doesn't tell you a lot, but it gives you a kind of framework. You can say, Gee, for anything that's as small as a drop of water sitting on wax paper, it's the contraction of its surface skin that is going to be the dominant force, so it's going to want to shrink into a sphere. When you get a much bigger drop of water and put it on wax paper, the dominant force is weight, gravity and so it's flattened out. There are lots and lots of changes you can observe in the kinds of things that exist, which are of necessity subservient to this principle. Like the little tiny animal which has to burn food mostly just to keep warm because the surface area through which he can lose heat is so big in comparison to his weight whereas a great big animal uses a much smaller fraction of his weight and energy to keep warm. There are just endless examples. You can't list them all but once you get in the habit of thinking in these terms you begin to notice.

That was a big challenge. Sally and I had supper together after class and we talked about some of the things David had said. "Perhaps it's OK not to understand everything," said Sally. "It keeps you wanting to know more."

In the last two classes teachers used all the materials we had collected for work with volume, trying to clarify questions about the relationships which David had explained. Shelley spent one class working with Sally and Harriet on the volumes of cylinders and cones. She wrote: "I was finally doing what I really wanted to do back in high school geometry. I had so much trouble with solid geometry and the sad part is that I don't think I ever even handled, much less investigated, a cone, sphere, etc. They were always drawings. So it was very exciting to discover the volume of a cone on our own." After the last Size and Scale class, Shelley reported:

> We decided to start making increasingly larger cones. I wondered how we could be certain that one cone would actually be double the size of the previous one. As we started I saw that if one linear measurement was increased a certain amount, then all the linear measurements would increase by that much, as long as the same shape was retained. I don't know why I never saw that before -- it's so obvious now... We saw that if the linear measure was 2X the original, then the volume was 2^3 and if the linear was 3X the original, the volume was 3^3 etc. so we predicted our next numbers and were correct. Amazing, what a good feeling!

Conclusion

It was good that for at least some teachers things came together in the end. The topic of the second teachers' seminar, light and color, seemed on the surface more difficult -- the physics of light are certainly not easy to grasp. But there were little understandings in almost every session of the course which were tremendously satisfying to the teachers. I think there were fewer frustrations in these classes than in the Size and Scale course, where teachers struggled for weeks with mathematical relationships without knowing where they would lead to. Teachers found themselves hopelessly confused about things like area and volume which they thought they understood, and they never really got a firm enough grasp of the geometry to begin to apply this knowledge to the real world where size and scale becomes meaningful and interesting. I think this class could make a great year-long course on geometry, biology, and physics, with size and scale as a unifying topic. In the eight weeks we spent on it, we barely got started. Yet inspite of the frustrations, when we asked the teachers if they wanted to continue the following fall for another semester's work, the majority were enthusiastic about returning.

Hedy was one of the participants who came back for the second teachers' seminar on Light and Color in the fall of 1981. The reader may remember Hedy's very first notes in January 1981 (see page 6) where she stated that she enjoyed hearing explanations to her questions on physics and astronomy but could never retain any of it. "It's like a fairy tale only I can't remember how it goes." In June 1981, at the end of the first teachers' course Hedy still did not have much confidence in her ability to understand scientific concepts. "I don't have a real belief that I could understand things," she said. "I don't reach out because I'm afraid I won't understand and then I'll be hurt." She told me that she took the class because "I decided to put myself in a position to be pushed... The content was incidental, though essential to the process which was me nudging up against the topics."

In the fall course on Light and Color, the content was no longer incidental for Hedy. After an individual session with David in December 1981, she wrote:

> I felt clearer coming into this class than I often have. There were several things I thought I really would like to know about. It was new for me to have a specific question in this area and to have some feeling of confidence that it will relate to other things I've learned. Like I just might be able to understand the answer to this question if I ask.

> I just realized how often I've asked questions about things I was curious about and maybe, at best, gotten an answer I enjoyed hearing, but I still didn't have enough of a framework to put the answer in or "keep" it. So it was pretty and perhaps I felt awe but I didn't feel the satisfaction I did after this class...
>
> I had some questions left (on electro magnetic radiation) but I felt so pleased to have put so much together that I didn't mind. I do feel more confident that given enough time and experience, lots more of these things will become accessible to me. That's new. I don't yet fully understand them but I think that I could. It's closer.

For a person who never understood scientific explanations, this represents a big leap forward. Hedy had moved from regarding explanations as fairy tales which she could never remember to feeling that topics of a scientific nature had become more accessible to her. In my essay on the Energy course, I wrote that the more I know about a subject the better I can accept open-endedness because the existing knowledge gives me the security to be left hanging. It's easier to tolerate not knowing when you feel sure that you could understand -- it gives you a sense of control.

Hedy concluded her remarks on her class with David with the following statement:

> I love having been let into this. My understanding is tiny but it seems like more than I've had my whole life. I have always been curious about astronomical things but my natural curiosity has been frustrated so many times by not being able to understand the answers I was given that I gave up wanting to know... I always seemed to need someone outside myself to help me focus on particular things to learn or ways to learn. Without that, I did not pursue many initial interests -- and there they all sit -- like so many unfired canons. I feel sad writing this in a way but also glad to be taking a look -- and very glad to have had the opportunity in this class to develop some new, real understanding. I think very often it is not clear what was missing until the gap is filled. In this case I feel as though I have never in my whole life had a satisfactory learning experience in science until now. And it is specifically and only because we have been able to go into so much depth and overlap the bits and pieces that I have ended up with not just little individual understandings -- like why red and green make yellow or how pinholes work, but all of it together that gives a larger picture of which those things are a part.
>
> I would love to go on indefinitely, meeting weekly, exploring new ground -- and no faster -- building up understanding slowly. That is certainly my pace. I would not have wanted to meet any more often. There's too much that's new to absorb any faster.

At the end of the second teachers' seminar we asked the participants for a written evaluation (see Vol. IV, pp. 238-279). Among other things, teachers were asked to comment on any continuing difficulties. Hedy worte:

> "Continuing difficulties" sounds negative. These dangling questions don't seem like difficulties - just more to know about. That's very positive for me I guess because I don't have to understand them right away. I'm under no pressure to unravel them.

Teachers were also asked to react to topics "left hanging" or inadequately explained, such as light waves. Hedy said:

> I don't feel frustrated by this. Motivated, curious instead. Saying that suddenly makes me cry because for me to even have and express such a thought is so brand new... I have simply never felt friendly towards those subjects.

The pleasure and excitement which Hedy expresses so well in her notes were shared by most of the teachers. Jean mentioned that she valued the respect that she was shown for her way of learning and thinking and how she enjoyed having the opportunity to explore, without fear, subjects about which she knew very little. Some teachers also were excited by their growing understanding of interconnections so that previously isolated topics which they had taught as science units were now seen as part of a large network of scientific ideas. To me, the most important single thing from the teachers' point of view was the access they were given to an exciting new world.

I doubt whether at the end of the second seminar any teacher would have asked the question which Sally posed after our first meeting: "How do they know...?" We may not have much more knowledge but we have learned that we, too, can understand. As soon as you have that conviction, you are no longer completely dependent on your teacher and the feelings of frustration and anxiety that often go along with such dependency become replaced by feelings of confidence, power and joy. That was the teachers' reward for the many hours of struggle during the course of the research seminars.

Section E

A PRELIMINARY TAXONOMY OF CRITICAL BARRIERS

ABRAHAM S. FLEXER

CONTENT

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E

A PRELIMINARY TAXONOMY OF CRITICAL BARRIERS

The original proposal to the Foundation offered to generate a taxonomy of critical barriers. That would have been an ambitious undertaking even for a three-year project of the scope requested. A project of the duration and scale actually funded could not hope to get that far. We did, nevertheless, accomplish a lot and made substantial progress toward developing such a taxonomy. A preliminary scheme is offered below, following some comments about the nature of the task.

A taxonomy of a set of elements reflects the theory that defines that set. Moreover, the relationships among a taxonomy, its underlying theory, and the elements they organize, are dynamic. Each element added to the set tests the taxonomy and the theory that shapes it. Each modification of the theory is tested by how well it accommodates the known elements of the set and by how fruitfully it predicts unexpected new elements that are subsequently found.

Taxonomies, like other theories, evolve. In its early form, a taxonomy is likely to be primitive, and the corresponding theory may account for observations more or less empirically, even speculatively. Mendeleev's chemical taxonomy and its underlying theory were of this sort. So were the biological taxonomy of Linneus and the underlying theory he used. During such formative stages, research is likely to be in the style of natural histories, in which observations and specimens are gathered and explored to see whether, and how well, they divide nature at evident "joints" (see D. Hawkins, 1966. Taxonomy and Information. Boston Studies in the Philosophy of Science, 3:41-55). At later stages, one deliberately seeks to test the generality of a maturing theory and to expand the useful range of its taxonomy. Research on maturing theories and taxonomies is more likely to take the form of testing strong inferences by tightly designed experiments performed under closely controlled conditions.

As this research program began, we had available a large body of observations relevant to critical barriers which had been collected over many years and in many settings, but neither a taxonomy nor a theory to organize them by. Those earlier observations had been guided in large part by the conviction that critical barriers are real and that careful observation would, in the end, yield useful insights. This natural history approach, analogous to that employed by the Challenger naturalists, has continued to be fruitful in ways described in David Hawkins' essay elsewhere in this report. Thus, during this research, we found what seem to be several useful joints at which our observations divide nature-- at least our sampling of it-- in fruitful ways. These reveal a number of categories of critical barriers which we list and discuss in this section. In a subsequent section, we list and discuss a variety of factors that make up the ecological settings in which critical barriers emerge, and which form a context for both teaching and research.

We find in our research three principle categories of critical barriers, each with several subcategories, and these constitute the divisions of the primitive taxonomy we propose below. We cannot yet assign each of our observations uniquely to one or another of these categories, for no strong theoretical basis for such assignments has yet emerged. Nor can we determine, *a priori*, whether these categories and subcategories are genera and species, respectively, or perhaps families and species whose genera remain unrecognized. Nor can we claim that the larger categories are mutually exclusive of one another as are the usual biological taxa, for example. These uncertainties should occasion no surprise, given the magnitude of the domain and our short foray into it. Nevertheless, the categories have already proved themselves useful and will become more so with further experience and reflection.

The categories, and the subcategories within them, are described below in some detail, and are amplified by examples drawn from this project's research and from other sources. The labels applied to each category and subcategory are necessarily cryptic and may mislead, as such labels sometimes do when interpreted too quickly at face value. Accordingly, we caution the reader to defer constructing a model of a category until the accompanying text and examples are read and analyzed. Given this caveat, we turn now to the first broad category of critical barriers.

PERVASIVE BARRIERS

Our analysis of the data revealed a category of critical barriers that were encountered in a wide range of contexts. These barriers emerged in so many different contexts that we suspect them to be (relatively) independent of subject matter. That is, although they are _recognized_ in connection with particular topics in elementary science, they may _operate_ at so fundamental a level as to affect a student's view of many other topics, perhaps the entire span of elementary science. It is in this sense that we apply the label _pervasive_.

Problems with Scientific Reasoning

Late in the course on _Light and Color_, participants were asked to predict where, in a wide mirror mounted on one wall of the classroom, various objects in the room would be reflected. As described in David Hawkins' essay, most participants employed a model in which reflections form on or near the surface of a plane mirror, and so their predictions generally were at variance with their subsequent observations. As soon as the discrepancy was noted, participants were invited to account for it. One participant asserted that the mirror "distorts the reflection depending on where you are" with respect to the reflected object. With some questioning, she expressed her model: the less the lateral separation between the observer and the object, the less the "distortion." When asked to explain how the mirror "knew" how much the lateral separation was, how it "knew" how much to distort on different occasions, the participant appeared not to understand the question. She saw no reason to be surprised at the need for mirrors in her world to "work" differently at different times: if her experiences with mirrors are different under different circumstances (less "distortion" of reflections of nearby objects), then mirrors *must* work differently at different times.

Most teachers of elementary science have encountered such behavior and many diagnose it as an inability to "reason scientifically" or to "isolate variables," or to "control" them (See L.C. McDermott, *et al*., 1980. Helping minority students succeed in science. *J. Coll. Science Teaching*, 9:135-40). We interpret such behavior as reflecting a more fundamental condition, namely, a student's acceptance of a world in which unfamiliar phenomena need not be lawful. The superficial lawlessness of the weather provides a commonplace precedent, as do more regular, but non-linear, phenomena such as exponential growth and gravitational acceleration. Accepting anarchy in the unfamiliar may condition a student's observations and experiences in ways that raise the threshold for perceiving even the outlines of regularity. Consider, for example, children using a stopwatch and tape measure to investigate the behavior of model cars rolling down an inclined plane supported by one or more standard blocks. Most students will recognize the need to time the descent of a car and to measure the distance it travels. Many will also recognize the need to replicate measurements, and some can apply the appropriate algorithm correctly. But when initial efforts fail to reveal some regular connection among time, distance and elevation (the number of blocks), interest in the investigation dissipates. With no expectation that the objects of study need behave lawfully, regularly, there is little incentive to continue past the point of confirming the unpredictability of the objects. Nor is there any conceptual framework within which to mount a concerted search for additional parameters, variables their teacher might say had been uncontrolled such as the initial position of the car on the ramp at each trial, or allowances for departures from straight paths, or the way initial and ending times were determined.

The lack of a commitment to a lawful universe may be among the most profound critical barriers students of elementary science encounter. With no notion of regular lawfulness, of theme and variation, there can be no recognition of unifying principles that simplify and rationalize experience and decrease redundancy in experience and knowledge. Each experience, each phenomenon is encountered as a new instance to be grappled with on its own terms, with little hope of being illuminated by past experience. Attempts to infer the regularity that underlies the rates at which spheres of different sizes and densities fall through liquids of different viscosities are likely to be stymied at the very outset-- by the failure to expect that there should be any regularity. Nor is this barrier restricted to children. Consider, for example, the bewilderment of participants in the Light and Color course as they grappled with the multiple and multiply colored shadows generated by a few projectors and theatrical gels (see Vol. IV, pp. 3-60 of the Raw Data)

Confusing Two Concepts that Apply to the Same Situation

Even when the possibility of an underlying lawfulness is acknowledged, other critical barriers may impede the growth of understanding. Consider the debate among youngsters comparing two simple pendula identical in all respects except their amplitudes. The debate centers on the question of which pendulum is faster. One side insists that the pendulum that moves through the larger arc is faster; the other maintains that because both pendula take the same time to complete a swing they are equally fast. Both sides, of course, are correct and the debate reflects a failure to recognize the multiple meanings of the term _fast_.

This simple example suggests a class of critical barriers that occur when the learner confuses two concepts which apply to the same situation. In the case cited above, the distinctly different concepts of _period_ and _average speed_ are overwhelmed during the debate about which pendulum is faster. An example about which many adults admit confusion involves kilowatts and kilowatt-hours. Both are measures of energy, but one is a measure of a quantity, the other, of power. Attempting to understand a phenomenon that involves either or both (for example, a utility bill) is likely to be frustrating unless the distinction is mastered. Moreover, the source of the frustration is often not recognized as deep and conceptual, as it almost always is in the latter example, in which the missing element is an understanding of the difference between a quantity and a rate.

A botanical example indicates how powerful such confusions can be. Many secondary students, newly introduced to the essentials of photosynthesis, can repeat the basics ("plants change carbon dioxide into sugar"), but are unable to explain the death of a plant kept in darkness for several weeks. Questioning generally reveals that, instruction aside, they regard the plant's _real_ source of nourishment as its roots, not its leaves. Roots do, indeed, take up water and indispensible inorganic nutrients, but this fact overwhelms an understanding of photosynthesis. The confusion is over the respective nutritional contributions of carbon dioxide and inorganic solutes, not over the role of photosynthesis.

A richer example, drawn from the _Light and Color_ course, involved a profound confusion between images formed by pigments applied to a surface on the one hand and images formed by patterns projected on a surface (e.g., from a slide projector) on the other [see The Hawkins essay in this report). Here, the missing element is an understanding of the variety of ways in which images can be formed and the consequent attempt to understand the less familiar projected images on the same basis as more familiar painted images. So long as that confusion remained in place, the participants were unable to comprehend the significance of images they observed in the _camera obscura_ or of the optical principles it demonstrates.

A final example suggests the variety of contexts in which this kind of pervasive critical barrier can be encountered. The context here is the explanations offered by elementary school children for the fact of the _weightlessness_ of astronauts. Explanations offered are often based on the absence of an atmosphere at the distances from earth at which objects become weightless. Once this inappropriate connection is made, an explanation becomes simple: with no atmosphere to press down on an object, it has no weight. Clearly, the confusion is not so much about the inverse square law, but about the very nature of weight and mass and the atmosphere's contribution to them. Again, attempts to intervene with instruction about gravitational fields will fail, because the student *knows* that weight is determined by the atmosphere. A similar set of critical barriers was documented as part of an investigation of young children's views concerning the nature of this planet (see J. Nussbaum, 1979. Children's conceptions of the earth as a cosmic body: a cross age study. *Science Ed*. 63:83-93).

As with other critical barriers, those in this subcategory will be difficult to detect and will be refractory to didactic instruction about the phenomenon under discussion. As usual, the barrier is not merely with the phenomenon, but in the conceptual tools available to the learner for dealing with the phenomenon.

Confusing Reality with Its Representations

The content of elementary science is such that representations are practically unavoidable. Inaccessible concepts, objects, and phenomena are routinely represented by verbal, diagrammatic, or mechanical representations. Genes on chromosomes are represented (analogically) by beads on a necklace, or modelled physically in clay or pipe cleaners. Photons are depicted as bullets, light waves are treated as if they were waves in air or water. Even advanced students of chemistry tend to think in terms of billiard-ball atoms connected by dowel-like chemical bonds. Cylinders are drawn as rectangles, cones as triangles, spheres as circles. Given that representations of these sorts are a ubiquitous feature of good pedagogy, it is no surprise that, for some students, some of the time, the representations become more real than what they represent. To the extent that students reconstruct their knowledge to fit more closely with the representation than with what is being represented, such representations may be unintentionally misleading. For a provocative debate on whether graphic illustrations are counterproductive in elementary science, see the transcript of K. Hammond's faculty seminar (Vol. I, pp. 100-123).

Generations of students of introductory genetics, for example, are convinced that chromosomes actually do break and rejoin in the manner of coils of clay, when in fact the mechanism remains a central mystery of the discipline. Similar distortions in favor of representation are common in chemistry (models of atoms, molecules, chemical bonds), in physics (the nature of light), and probably in every context in which representations are important pedagogical tools. But these tools can also inhibit learning, can, we feel, constitute critical barriers to learning. Consider, for example, the following statements by teacher-students in the Light and Color course:

> I think in general for me one of the problems I have in dealing with some of these topics [wave-particle duality] is that my mind becomes fixed on some representational image, which is not necessarily accurate.
>
> Polly, notes of 11/18
> Vol. IV, p. 103

> "White light" seems to be a misnomer-- really means clear light, natural light.
>
> Hedy, notes of 10/7
> Vol. IV, p. 40

For another example, from this course, of interference between representation and reality, see David Hawkins' description of the demonstration of how shadows form. See also Ron Colton's discussion of learning problems associated with diagrams and sections.

Note, however, that the warping of reality to fit a novel representation may be highly productive when skillfully done with the student's understanding. For example, most adults are unable to use only a syringe to fill an "empty" jar inverted in a pan of water. A typical attempt involves filling the syringe with water and trying to add its contents to the jar. Few persons know what to try, once this approach fails. But when a skillful teacher intervenes with the model of an equal arm balance (one arm being the atmosphere, the other the air in the jar), many will delightedly fill the jar by using the syringe to withdraw air from the jar. Our point here is twofold: that representations are not to be avoided merely because they may be counterproductive; and that many students do confuse reality with representation, and do so often enough for us to be convinced that such confusions generate an important class of critical barriers.

Two subcategories of pervasive critical barriers remain to be described and documented. Like barriers in other categories, both occur when learners attempt to draw upon existing stores of information and knowledge.

Making Inappropriate Associations

Another part of this report, David Hawkins develops the metaphor of mind-as-filing-system to examine the ways a person might store and retrieve her or his fund of information and knowledge. It was suggested there that, when confronted with a novel situation, one searches one's memory for a "file" with a title (or content) that seems relevant in some meaningful way. When all goes well, the match between the file retrieved and the situation at hand is fruitful. To reuse an earlier example, a person who regards light as something that "travels in straight lines" is able to regard files about plane geometry as relevant to a variety of optical phenomena.

But things do not always go well. Someone who has no files about plane geometry may be unable to cope with the location of images in plane mirrors, with a *camera obscura*, or even with colored shadows cast by light filtered through colored gels. For him or her, observations of such phenomena may remain transiently meaningless, unconnected and unconnectable with prior knowledge because no relevant file exists to receive or inform the observations.

Even when a relevant file exists, one may be unable to retrieve it in a particular context. Most adults, for example, are aware of a pervasive atmosphere of gasses that generate a force familiar to them as *air pressure*. Most adults are also aware of the laws of simple balances. Yet few adults can retrieve those files as relevant to the challenge, mentioned above, of using a syringe to fill a jar inverted over a pan of water. A carefully constructed metaphor (a "balance" consisting of the atmosphere on one side and the air in the jar on the other) may illuminate the situation by making both (and possibly other) files easily accessible.

As experienced teachers know, provocative analogies of this kind can be powerful pedagogical devices. By bringing into focus some facet of a situation or phenomenon not earlier obvious, they enable the learner to select from among the mental files the few that may be relevant in ways not previously recognized. In this sense, analogies help to transform one's perceptions in fruitful ways. Piaget's's *schèmes* serve similarly to transform perceptions. Once the transformation is made, entirely new information and knowledge may be retrieved and brought to bear on the situation at hand, as in filling the inverted jar or comprehending a *camera obscura*.

But when no provocative analogy, no germinal schème, is available, knowledge already possessed by the learner may remain occult, inaccessible, useless. The most powerful of filing systems is useless without an effective index which can be scanned rapidly and conveniently, and the inability to scan one's own mental index may comprise a correspondingly powerful critical barrier.

Retrieving Malinformation

This final category of pervasive critical barriers complements the previous one. Instead of failing to locate a file relevant to the immediate context, however, the learner retrieves as relevant a file that contains information that is factually incorrect and misleading. We have in mind here more than mere misinformation, more than incorrect details such as a mistaken melting point or an inaccurate conversion factor. Although inconvenient and temporarily misleading, the impact of such misinformation is rarely great and these lapses hardly constitute critical barriers.

Rather, we have in mind forms of misinformation that function as theories, models, Piagetian schemes. Just as factually correct schemes facilitate powerful transformations of perceptions, so, too, do factually false schemes. And just as correctly transformed perceptions can be enlightening, incorrectly transformed perceptions can be damaging, as described below. To these factually incorrect schemes, and to theories and models that distort in similar ways, we apply the label malinformation. Transformations based on malinformation can be so misleading that we regard the retrieval of malinformation as a form of critical barrier.

Consider, for example, the following statements about the nature of light written by participants early in the course on Light and Color.

> ... think I knew shadow is the result of absence of light, but have always thought of a shadow as something more positive-- more like an imprint, or as in a photograph.
>
> Polly, notes of 9/23
> Vol. IV, p. 18
>
> Does it [light] travel? I don't think of it as something that moves. I think of it as just being there where ever it is and there is more or less of it.
>
> Hedy, notes of 9/23
> Vol. IV, p. 7

These participants, and others like them, used the contents of files they regarded as relevant to the topic of light to transform their experiences and observations-- with predictable consequences. Because they mistransformed their experiences, using malinformation, they encountered considerable trouble understanding colored shadows and the effect of colored gels on light projected through them. These troubles persisted for weeks, even after concerted attempts to convey conventional _scientific_ explanations. Their common sense, personal theories were tenaciously preserved, even in the face of dissonant experiences-- just as one would expect such theories to be treated. As expressed in a related context by one of the participants quoted immediately above,

> I also felt really interested in why light behaves so differently from pigment and pigment is all I have to relate to and that's where my common sense is and I felt like [a staff member] was saying, "Just don't think the way you think." I feel like I can't let that go, that's all I know about.
>
> Hedy, notes of 9/30
> Vol. IV, p. 179

Items of malinformation familiar to most teachers of elementary science include the notions that there are two thermal fluids (hot and cold); that light has little to do with color; that plants get most of their nourishment from the soil; that matter is solid (i.e., free of the voids required by atomic theory); and so on. These personal theories inevitably conflict with the conventional view proffered by elementary science. Continuing to quote the same person:

> ... when as a student, I step off into a whole realm of questioning then I don't know if such lines of questioning are in the right direction or not. Our naive or commonsense questions sometimes are in the

> opposite direction from understanding scientific truth. So when many explanations are far-fetched, or I am trying to understand far-fetched things, I am depending on a teacher to help point me down the fruitful path.
>
> Hedy, notes of 11/14
> Vol. IV, p. 96

Because correcting such malinformation requires reconstructing many individual files, and also possibly risks having to reconstruct large parts of the filing system itself, didactic instruction in the facts is unlikely to be effective in dispelling critical barriers that result from retaining the malinformation. High levels of cognitive dissonance, preferably induced by personal experiences, will be more effective. Experienced teachers are aware of this resistance and of effective ways to help students to overcome it.

These, then, are the five subcategories we propose in the larger category we have called pervasive critical barriers to the learning of elementary science.

Problems with scientific reasoning
Confusing two concepts that apply to the same situation
Confusing reality with its representations
Making inappropriate associations
Retrieving malinformation

We turn next to the second large category of critical barriers.

BARRIERS RECOGNIZED IN NARROWER CONTEXTS

This second major category includes five subcategories of critical barriers that seem from our sampling to be less profound, and to have less generalized effects, than those included in the first major category. It is one of two categories of less pervasive critical barriers that we propose in this preliminary taxonomy, the other being barriers related to mathematics (see below). Although less pervasive, these are still critical barriers; that is, they result in broad misunderstandings, are resistant to didactic instruction, and are overcome when learners reconstruct their knowledge and thereby acquire powerful new understandings with wide application.

The taxonomy of this category is more provisional than that of the previous one because we are less certain about the status of each element and the subcategories to which they are assigned. Additional experience and analysis may reveal, for example, that a particular element is less resistant to instruction and to require less reconstruction of the learner's knowledge, in which case we might no longer regard that element as a critical barrier. Or, an element proposed below might be reinterpreted as a species of a more pervasive critical barrier, either one already proposed above or, perhaps, a new genus or family. In either case, that element would have to be removed from this list. On the other hand, two or more subcategories proposed here may later be seen as instances of a more general subcategory, or a single subcategory may later be seen as an inappropriate combination of two or more previously unrecognized subcategories.

Although the taxonomy proposed below is provisional, it does represent our current best construction of the data and our analysis of them.

With these caveats, we can proceed to develop these subcategories and their elements, drawing on our experiences of seeing many learners of many ages encounter similar-- homologous, we think-- difficulties in superficially different contexts. These difficulties cluster in a way that suggests to us the following five subcategories as principal contexts for this kind of critical barrier:

invisibles and impalpables;
conservation laws;
wholes and their parts;
constructs and their measures;
scale and relativity.

As will become clear below, overcoming the critical barriers in these subcategories requires the learner to formulate novel, germinal insights. Learners who have formulated such insights find them to be illuminating, liberating. Learners who have not yet formulated such insights remain constrained, especially in their interactions with teachers and texts that assume the insights to be in place and usable. Yet the kinds of insights required in the contexts described below seem less global, less profoundly fundamental than those we defined earlier as pervasive barriers. True, formulating these insights does require the the learner to reconstruct her or his knowledge, but in these cases, the reach of both the constraint and the liberation seems less. We nevertheless feel that these are critical barriers, and so merit special attention from teachers and researchers.

Invisibles and Impalpables

Instances of the first subcategory of these less pervasive critical barriers stem from a learner's inability to deal with a construct that is, to the learner, either invisible, or impalpable, or both. Most students of elementary science have trouble, often prolonged, conceptualizing such invisibles and impalpables as: the atmosphere; atoms, molecules, and chemical bonds; subatomic particles, including electrons and their orbitals; electrical currents and magnetic fields; cells and their organelles; and energy. These and many other entities have become sufficiently real to the advanced student to have become visualizable, but to the beginner they are so novel, so mysterious, as to be conceptually inaccessible. We refer here not just to difficulties dealing with one or another construct that is invisible or impalpable, but to an inability to deal with an entire class of such constructs in elementary science.

Consider, for example, the conventional view of the atmosphere as a mixture of gasses, each of which has specific properties, including mass. A person who has fully appropriated that view and its ramifications will probably regard objects such as barometers, manometers, aspirators, pneumatic troughs, even siphons and drinking straws as easily understood. More important, these objects and many others will be understandable as manifestations of the same physical entity (air), the same conceptual framework (material gasses). A person who does not yet perceive a material atmosphere, by contrast, can't comprehend these objects and can see no connections among them. These

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objects are not only not understandable as a class, but each is individually mysterious, unrelated to the others and to any organizing framework. This is why we perceive a critical barrier here. To appropriate the conventional view, a learner must reconstruct not only his or her experiences and knowledge about air, but also about gasses in general, and about underlying similarities among previously unrelated objects and processes. Once the reconstruction is consolidated, an even greater range of objects and processes becomes easily understandable in the light of the new insight. As David Hawkins argues in his essay, neither the reconstruction nor the consolidation is likely to come about through didactic instruction about the physics of gasses. As most teachers of elementary science know, direct instruction on such topics must be supplemented by a variety of demonstrations and direct experiences (laboratory "experiments") and must await the student's intervention to effect the reconstruction.

Much of the domain of chemistry-- atoms, electrons, chemical bonds-- is invisible and impalpable in an even deeper sense. Air and the atmosphere can be perceived, for example, as the contents of an inflated tire or as wind, respectively, in ways that atoms, electrons, and chemical bonds cannot be. One can, with some justice, claim that much of the content of an introductory chemistry course is (or ought to be) devoted to encouraging the beginner to reconstruct his or her experiences with material objects to become consistent with the conventional view that all matter is aggregations of unimaginably small units of only one hundred or so different kinds. Teachers of elementary chemistry

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have, over the years, accumulated a set of demonstrations, laboratory experiences, and other pedagogical devices that *appear* to teach these aspects of the chemical view. We argue, however, that their students learn, when they do, not just by being taught, but by actively reconstructing their knowledge. Further, we argue that elementary chemistry is difficult to learn precisely because the reconstruction being required involves entities that are invisible and necessarily remain difficult to conceptualize until after the reconstruction has been achieved.

Energy, at least in the conventional thermodynamic sense, provides another instance of a critical barrier that emerges in many contexts and so belongs in this subcategory. Energy is variously spoken of as: kinetic or potential; mechanical, thermal, solar, or nuclear; biological, chemical, or physical. These modifiers testify to the importance of energetics and thermodynamics as an organizing principle across disciplines and across topics within disciplines. Yet the constructs designated by these modifiers are difficult to render visible for the beginner, except indirectly. Beginners must therefore work at reconstructing their knowledge on the basis of experiences with warm and cold objects and with a handful of examples, metaphors and analogies (winding springs, moving boulders uphill) not all of which are are useful.

Examples of the difficulties of appropriating a conventional view of heat and temperature were described and analyzed by David Hawkins. Notions that heat *flows* and that it is passively conducted help some learners, but not others who require active transport of heat by conductors. Similarly, the metaphor of electrical currents being water-like helps some but not others who cannot accept the notion of flow through a "solid" conductor.

Difficulties with energy and energetics may stem, at least in part, from other critical barriers which exacerbate those that stem from the invisibility of the constructs. Quantities of energy are frequently measured and expressed as ratios, in odd units, and as the consequence of one or another transformation (burning, melting, etc.). All of the latter entail critical barriers quite independent of those directly connected with the invisibles of energy.

The course on Light and Color revealed that, in an important sense, many learners regard light as invisible until it is perceived at the retina.

> How can you describe something [i.e., light] that you can't see (well, not really), touch, smell, etc? Well, you do see light, or rather you need light to see. So light becomes related to VISION: LIGHT PLUS EYES MAKES VISION POSSIBLE.
>
> Maja, notes of 9/23
> Vol. IV, p. 10

On this invisibility hinged the crucial reconstruction of light as something that "travels" in straight lines. Until the class could deal with (to them, invisible) light rays and beams, they were unable to come to grips with the geometrical interpretation of optical phenomena and were foreclosed from a powerful reconstruction. To reuse a passage quoted above in another context:

> Does it [light] travel? I don't think of it as something that moves. I think of it as just being there wher ever it is and there is more or less of it.
>
> Heddy, notes of 9/23
> Vol. IV, p. 7

Elementary science pushes the student into many encounters with invisibles and impalpables. In addition to those cited, we suggest cells and their organelles, the biological counterparts of molecules and atoms; electricity, magnetism, and gravity; even geometry, where students are asked to come to grips with form disembodied from matter. There are enough of these, and they are different enough, to convince us that problems dealing with invisibles and impalpables constitute a set of critical barriers that belong in this subcategory of semi-pervasive barriers.

Conservation Laws: Transformations and Cycles

Another cluster of learning problems emerges when students of elementary science are inappropriately expected to become committed to the conservations laws. Conservation laws in this connection are not those of Piaget, although those may be importantly involved. Rather, we refer to the principles that energy and matter are (inedpendently) conserved through all transformations carried out under non-relativistic conditions. These principles are crucial to modern science. Indeed, Lavoisier's exploitations of early intimations of the conservation of matter mark the beginnings of modern chemistry. With no commitment to these principles, much of elementary science becomes senseless, yet many learners of all ages have no such commitment. An example will suggest a set of failed understandings-- critical barriers-- that can result when these principles are not strongly held.

"Bottle gardens" were, for a long time, popular displays at the Mountain View Center. Some of these contained the usual combinations of earth, air, water, and plants; others contained pieces of fresh fruit, baked goods, or other foods. In its most provocative form, a bottle garden was tightly sealed and suspended from one arm of a balance which was then carefully equilibrated. A nearby sign described the apparatus, drew attention to the tight seal, and invited the observer to predict the future tilt of the balance's arm.

Without a belief in the conservation of matter, many outcomes are plausible: a garden may "gain" weight as plants grow, or may "lose" weight as water, soil, and air are used up; a garden may gain weight as molds flourish on the foods, or lose weight as the food is reduced to a grey sludge. With a belief in the conservation of matter, only one outcome is possible if the garden is truly sealed. In one case, interest centers on deciding what might be the dominant process process(es) in each garden; in the other, interest centers on how well each garden is sealed, on the source of materials that "become" the larger plant,, on the fate of the materials that "were" the orange.

As with other kinds of critical barriers, the ramifications of this kind of malconstruction are diverse. With no commitment to conservation laws, chemical reactions are likely to be misunderstood. An acid neutralized by a base no longer exists, even though it seems replaced by something salty. There is no compulsion to connect the products of the neutralization with the salt. Burning coal or wood becomes smoke, ash, and, perhaps, some water vapor. There is no need to search for a colorless gas among the products. There is no reason to seek connections among the chemical transformations of the kinds involved in the carbon cycle (photosynthesis by plants as the complement of respiration by animals), or among the physical transformations of the water cycle (evaporation as the complement of precipitation). These sorts of transformations need not be linked in any significant way. Indeed, talk of *cycles* is likely to be thoroughly puzzling.

Similar misunderstandings arise when the conservation of energy is not established as an integrating principle. Wet and dry cell batteries *make* electricity for a time, then *wear out*; only some kinds can be *recharged*. Nuclear reactors, windmills, and photovoltaic units can *generate* electricity, but solar panels cannot. Photosynthesis *traps*, and respiration *releases* the sun's unlimited energy, and somehow plants and animals both benefit. In the absence of a conservation law to help organize such data, they remain unrelated, even disconnected.

Wholes and Their Parts

A suspension of cells, say unicellular green algae, appears to the unaided eye as a relatively uniform, pale green liquid. When diluted serially, the color becomes progressivley paler. After many serial dilutions, a water-clear suspension is obtained; the algae can no longer be perceived with the unaided eye. A few drops of the suspension spread on a solid culture medium will, in a few days, produce a number of discrete, green colonies. Any colony can be removed and suspended in water to reestablish a suspension much like the starting suspension.

These techniques are the foundation of modern microbiology, and the approach has important places in population biology, chemistry, and elesewhere. Yet the underlying logic baffles most beginners. For many students, the confusion arises when they mistakenly visualize the algae (or at least the source of the green tint) as continuously distributed throughout the suspension, ignoring (or unaware of) the fact that algae are particulate. Half of a drop that contains one hundred cells will contain about fifty cells; but half of a drop that contains a single cell has a reasonable chance of being empty.

Learners overlook the microscopic discreteness of the components of macroscopic aggregates in many other contexts. Indeed, this kind of misperception is so common that we believe it to underly a set of critical barriers nicely subsumed under the rubric of confusing the parts and the whole. (These problems may be connected with, perhaps compounded by, critical barriers that derive from the inability to deal with invisibles and impalpables, as described just above. A solution of salt water is not homogeneous, but a is collection of discrete, albeit very small, entities. So, too, is a bit of tissue an aggregate of individual cells. Students of chemistry and biology who confuse these wholes and their parts are likely to find many aspects of these sciences inaccessible. A student who perceives salt water as homogeneous cannot comprehend the details of osmosis, nor can a student who regards muscle tissue as homogeneous comprehend the mechanism of contraction. Nor will instruction on the thermodynamics of equilibrium or on the molecular architecture of actinomyosin overcome the misunderstanding.

Those latter ideas make no sense when applied to entities (mis)perceived to be homogeneous. The attendant difficulties lie deep, in conceptions of wholes and their parts. Students are not likely to resolve their problems until reconstructions are achieved that recognize the fine-structural discreteness of superficially uniform aggregates. Until that kind of reconstruction is managed, critical barriers will be encountered in many settings. This confusion of parts and wholes may underly much of the common difficulty encountered in approppriating the details of atomic theory. The relationships among atoms, molecules, and macroscopic objects are likely to be obscured by these. So are the connections among cells, nuclei, chromosomes, genes, and nucleic acids. It is because students of elementary science encounter so many entities of this sort, entities that are superficially homogeneous but microscopically discrete, that we regard this difference to be the source of an important subcategory of critical barriers.

Constructs and their Measures

Another cluster of generalized critical barriers flows from confusing constructs with their measures. The confusion between heat and temperature, already discussed in David Hawkins' essay in this report, is typical of this set of difficulties. Recognizing the distinction between the construct (heat) and its measure (temperature) grants access to the understanding of phenomena and processes that might otherwise remain unapproachable. For example, the notion of extracting heat from the winter ground or ocean must seem bizarre to a person who knows those bodies as cold, i.e., of low temperature. Such familiar machines as refrigerators, ice-makers, and air conditioners are likely to remain mysterious to those who have yet to accomplish this essential distinction.

The distinction is not likely to come from instruction, for, once again, a reconstruction of knowledge is the key. It is more likely that experiencing the effect of dry ice on liquid nitrogen will engender the reconstruction, As was the case in the course on Heat (refer to David Hawkins' account of that course.) Once established, other thermal phenomena become understandable as a class, including the several heat pumps cited above.

This sort of confusion is sufficiently widespread to have resulted in at least one concerted nationwide effort to eradicate an important subset of them. Recall the attention given by mathematics curricula such as The School Mathematics Study Group and the University of Illinois Committee on School Mathematics to the distinction between geometrical constructs (angle, line) and their measures (degrees, inches). Indeed, an entire generation of teachers and their students acquired the habit of saying, "The measure of the angle is forty-five degrees" without, apparently, comprehending either the important distinction being offered or why the distinction might be worthwhile.

Other examples of this sort of confusion occur in connection with acidity and pH, and with electricity and its measures. Even more common is the misunderstanding of the differences between massiveness (materiality, amount of matter) on the one hand, and mass, on the other. When in place, the distinction provides the basis for rationalizing derived constructs such as density and specific gravity. Without the distinction, these derived constructs generally remain opaque, as anyone who has attempted to teach them is aware. A similar cluster of confusions is connected with notions of action and rate. These confusions make difficult such disparate topics as ballistics, hydraulics, and titrations.

In each case, the confusion has the potential to generate critical barriers in many other specific contexts which may seem otherwise unrelated. None of these confusions is likely to be dispelled merely by instruction, the examples of mass and density being only archtypical. But, once acquired by the learner who appropriately reconstructs her or his knowledge, the distinction confers wide ranging new insights. It is for these reasons that we perceive here a cluster of critical barriers.

Scale and Relativity

A final subcategory of these less pervasive critical barriers involves failures to appreciate, or to recognize , the consequences of radically changing the scale of an object or phenomenon. These barriers include, or are tangential to, others that pertain to the capacity to reason relativistically (see Ron Colton's essay on critical barriers related to mathematics). A few examples will establish the central issues.

Most people recognize that important features of common objects and phenomena depend on their scale. Most people will recognize that a pencil ten times larger than ordinary, or ten times smaller, is not likely to be routinely useful. However, few persons realize, and many would be startled to find, how much of the utility is lost across such enlargements and reductions. A tenth-sized pencil approaches the dimensions of an hypodermic syringe, and a ten-times larger one would be unwieldy as a bseball bat! Relief maps are almost invariably misconstrued, even though they carry notices that the vertical scale has been exaggerated "for clarity." Few people realize by what factor, for most are astonished when told that the tallest U.S. mountains (nearly three miles high) would be represented by bumps about one millimeter tall on a map that represents the 50 contiguous states as one meter from coast to coast.

Our point is that few learners are able to make even reasonable extrapolations of the consequences of expanding or shrinking objects and phenomena over many orders of magnitude. Yet elementary science includes many topics that are likely to remain incomprehensible to learners who have not acquired that ability. For example, general science classes, beginning in the upper elementary grades, introduce atoms, cells, planets, and galaxies as more or less equivalent kinds of objects, with no suggestion of the important differences connected with their respective scales. Indeed, the solar system is commonly (and misleadingly) used as an analogy for the Bohr atom.

The consequences of this inability to comprehend the significance of large differences in scale are many. Few people realize, for example, that our planet is proportionally smoother than an orange, or that electrons behave in ways impossible for macroscopic objects. Of the thousands of students introduced each year to the compound microscope, few ever come to comprehend the relationship between the actual size of a cell and the cell's optical image. A similar misperception seems to apply to celestial objects viewed through telescopes. Finally, even the few people who recognize the uniform, thousand-fold multiplicative steps progessing from one to one thousand to one million to one billion, are generally unaware that the arithmetical increments are 999, then 999,000, then 999,000,000.

The kinds of critical barriers that result from these misunderstandings were more thoroughly documented during the course on Size and Scale, offered as part of this research. This is not the proper place to report the data gathered during that course, or to analyze them. At this point, we can only end with the assertion that the inability to comprehend the consequences of large changes of scale leads to other critical barriers in many other contexts. And we again assert that mere instruction is not likely to generate the kind of intellectual reconstruction required to acquire the skill and the empowering insights it confers. The reader interested in the data and interpretations beneath these assertions is referred to the other essays included in this report.

BARRIERS RELATED TO MATHEMATICS

It is reasonable to expect that many critical barriers encountered by learners of elementary science are connected with deficiencies in mathematical skills and insights. That is indeed the case. Indeed, our findings in this regard were so rich and so diverse that we devote an entire section of this report to describing and interpreting our experiences (see Ron Colton's essay, *Problemswith Mathematics*). At this point, we merely list some of the topics discussed there.

- Geometry
- Permutations, combinations, factorials
- Ratio and proportion (density)
- Units, conversions, interconversions
- Nonlinearity (includes reciprocals, exponentials)
- Relative motion
- Simple calculations
- Selecting among algorithms

DISCUSSION AND IMPLICATIONS

These, then, are the kinds of critical barriers that emerged from our analyses of observations made during this project and from other sources. We have assembled them into a tentative taxonomy (Table 1). The listing is not exhaustive, nor are the categories and subcategories likely to persist unchanged. But this is a start, and it may provide a framework for subsequent research on teaching and learning.

TABLE 1. PROPOSED TAXONOMY OF CRITICAL BARRIERS.

- Pervasive Barriers
    - Problems with scientific reasoning
    - Confusing two concepts that apply to the same situation
    - Confusing reality with its representations
    - Making inappropriate associations
    - Retrieving malinformation
- Barriers Recognized in Narrower Contexts
    - Invisibles and impalpables
    - Conservation laws: transformations and cycles
    - Wholes and their parts
    - Constructs and their measures
    - Scale and relativity
- Barriers related to mathematics

The next task is to test this taxonomy against additional experience with other students in other contexts. It will also be important to compare the inferences from such studies with parallel investigations of the ways in which people reconstruct their knowledge (see, for example, the transcript of K. Hammond's seminar, Vol. I, pp. 100-123). We suspect, as is often the case in the human and social sciences, that the interaction terms will be more important, or at least more informative, than the individual factors. In this case, the relevant interactions are likely to be among the critical barriers and the situations in which they emerge. For a preliminary analysis of those situations, see the essay, on the Ecological Contexts of Critical Barriers, elsewhere in this report.

It is our hope that we have managed to reconstruct current views of parts of pedagogy, of the psychology of learning, and of techniques for studying both. We further hope that the reconstruction, if validated, may overcome a critical barrier that underlay decades of curriculum development. That barrier seems to have been that changing the *content* of curricula was the principal requirement for improving learning. If the reconstruction implied by the present work is valid, it will emphasize the need to recognize the differences between *teaching* (conveying information) and *learning* (reconstructing knowledge). It is our conclusion that changes in curriculum may improve the packaging of information but will not alone facilitate learning.

Mountain View Center

Research on Critical Barriers,

Fall 1980 - Spring 1982

Problems with Mathematics

by Ron Colton

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PROBLEMS WITH MATHEMATICS

Some General Difficulties

a) Whatever the nature of individual difficulties, most of the students in the Energy courses were not able to think of everyday situations in quantitative terms. Mathophobia was prevalent. This term does not mean particular difficulties in understanding subject matter, or inability to calculate --though both problems were abundantly evident-- but, as the term implies, being afraid of numbers. Some students "turn off" almost automatically when numbers appear because numbers create a sort of mental paralysis. There is an attitude of despair: "I cannot understand figures and I never will", even though these students may use figures tolerably well in their daily lives. Mathophobia is far too widespread a problem to be commented on in depth here, save to say that mathematics is looked on as a classroom subject that is not valued or understood as a tool in the real world. Unfortunately, many of the so-called "realistic" problems that students have had to confront are anything but realistic. 527 ÷ 54 does not make any more sense nor seem any more relevant for most people when it becomes 527 oranges shared among 54 children. Despite the fact that throughout the Energy courses instructors were careful to make sure that the figures in their examples were realistic, some students felt that somehow their answers might be correct even if they did not make sense in everyday terms. So students, given a problem about the cost of cooking a turkey, where the correct answer was 82.5¢, put down in one case $825.00 and in another $825,000.00, because the figures came out that way!

(b) Students in general were afraid to estimate or to round off numbers. They were disturbed by ball park figures which instructors frequently used. Likewise they were dissatisfied with the results of their own experiments when the results, while of the right order of magnitude, differed from the "correct" answer.

It seems that we like to attach definite labels to ideas, to fit them into specific "files". Anything that upsets this system, that does not fit into a file, is disturbing. Thus, biological classification is relatively easy when there is a series of distinct species; it is the subspecies and hybrid swarms and other variants that make things difficult for the biologist. Our elementary mathematical education is generally concerned with the absolutely right, so that 107986 as an answer may be right and 107987 wrong, though in most practical situations the difference doesn't matter and the precision with which the data were collected probably doesn't justify the distinction.

The following problems, using supposedly realistic situations, are taken from a university study*, financed with Federal funds. They illustrate how impractical such examples can be:

Metric Units of Area

"1. The top surface of a particular wood stove is 55.88 centimeters wide and 88.9 centimeters long. What is the area in centimeters of this surface?

2. A second stove is 5588 millimeters wide and 8890 millimeters long. Find the area of the top surface of this stove. Give your answer in square centimeters.

Metric Units of Volume

1. A cord of wood is a stack 1.216 meters high, 1.216 meters wide, and 2.432 meters long. What is the volume of a cord of wood? Give your answer in millimeters.

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* A Sourcebook for the Design of a Regional Environmental Learning System. Volume 6. Content - oriented resources. University of Virginia, School of Engineering and Applied Science.

2. A face cord of wood is a stack 1.216 meters high, .608 meters wide, and 2.432 meters long. What is the volume of a face cord of wood? Express your answer in millimeters.
3. A certain woodstove is 9 decimeters long, 3.75 decimeters wide, and 6.75 decimeters high. What is the volume of this stove? Give your answer in millimeters."

(Pages 110-113 of the Sourcebook)

The dimensions are given to an implied degree of accuracy that is obviously meaningless - the length, breadth, and height of something as irregular as a cord of wood to the nearest millimeter, with an answer required that requires the volume of some 64 cu. ft. of logs to an accuracy smaller than an average grain of sawdust!

Furthermore, in the first question, area is asked for in centimeters, in the second in square centimeters, while volumes in the remaining three questions are asked for in millimeters.

Another example:

> "2. Warmth. These cuttings root best at temperatures between 15.5° C and 26.7°C. Results are usually best if the soil is warmer than the air. To obtain this 'bottom heat' we suggest using heating elements under the soil in the propagating case (see page 216)."*

In this quotation the common error of implied precision occurs yet again. The intention was to suggest a broad and noncritical temperature range (60°-80°F). The editors, in converting to the metric scale, accurately transformed the figures but drastically changed the meaning.

(c) Ratios of all sorts present difficulties. Sometimes the ways in which the term ratio is used is confusing; for example, high gear might be $\frac{1}{2000}$ and low gear $\frac{1}{50}$ but large scale maps might be $\frac{1}{500}$ and small scale $\frac{1}{20,000}$. Ratio and proportion problems almost invariably confuse students. The concept is more difficult than the straightforward relationship of A = B or A = 5B or ½B. "A varies with B" or "A is proportional to B" has an uncomfortable air of indefinitemess about it. Likewise, "triangle ABC is similar to DEF" implies that there is a vagueness inherent in the statement and some computation to be done before the critical answer "how big" is to be found. If the indefiniteness of the situation is the problem, or part of it, then it probably falls into the category of difficulties which includes estimation, approximation, using ball park figures, and perhaps nonlinear relationships. If A = 5B, one can visualise the problem as addition - five objects equated with one, or if A = 1/5B the same sort of visual image holds. With more complex relationships, one cannot see the problem in terms of simple adding-to or taking-away-from.

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* Nuffield Junior Science. Teachers' Guide "Plants and Animals" SRA Collins, 1967.

There is a lack of concreteness in many ratio and proportion problems which makes them hard to tackle. "Mix 6 cups of flour, 3 sugar and one of butter" is an instruction that is easily followed but "mix flour, sugar and butter in the proportions of 6, 3 and 1" is often not understood, partly because there are no concrete units and it is not obvious to many learners that the units don't matter.

A very common kind of calculation in elementary science (as in everyday life) is of this type: 150 ccs water dissolved 10.7 grams of a salt. How much is this in terms of gms/liter? Even when illustrated in such everyday terms as, "a six pack of beer costs $1.20, how much would 10 cans cost?" this poses problems. Students cannot perform this task in terms of unitary method which, if a little cumbersome, at least proceeds in logical, understandable steps.

6 cans cost $\$1.20$

So 1 can cost $\frac{\$1.20}{6}$

So 10 cans cost $\frac{\$126}{6} \times 10$

Ratios and proportions turn up in many other situations and are so frequent a cause of confusion that we have included them as an item in our "Matrix of Critical Barriers". One particular case, mentioned here because of what follows later in connection with spatial concepts, is the concept of similarity and in particualar of the relations between the sides of similiar triangles. If this concept is not grasped, then the fundamentals of trigonometry and those skills based on it, such as surveying and navigation, cannot be thoroughly understood.

(d) There was often a lack of a "factorial sense" so that students failed to see the relations between, say, 8, 32, 64 and so carried out needlessly lengthy calculations.

(e) A number of students had difficulty with the formal logic of, "if A = B and B = C then A = C."

(f) The use of averages created some difficulty. How can an average family own 1.7 cars?

(g) Perhaps related tc the above, how can one define a quantity in terms of *hours* if only a few *minutes* have been involved, for example, if a 60W lamp burns for 25 *minutes*, how can you have an answer in Watt *hours*?

(h) Specific difficulties apart, a number of students were slow and uncertain in carrying out elementary arithmetical calculations, while fluency in mental arithmetic was very rare indeed. This is particularly unfortunate because mental arithmetic deals with *numerals*, written arithmetic with *figures*.

(i) Conversion of degrees Celsius to degrees Fahrenheit and vice versa. The general difficulty is widespread and well known. However, a number of students, struggling with these conversions, correctly answered 100 and 180 respectively when asked, "How many steps between freezing and boiling on the Celsius and Fahrenheit scales?" Then they said that each of the 180 steps would be larger than the hundredd steps, though the total "distance covered" in each case was the same.

While this is a purely numerical problem, it became a spatial problem because, in order to try to help the students, it was represented in spatial terms; "Imagine two ladders each standing on the same base and reaching the same height. One has 100 steps, the other 180... Which are the larger steps?" Had this been an isolated case it could have been overlooked as a student's momentary lapse, but the fact that several students made the same mistakes was surprising and interesting.

(k) Permutations, combinations, factorials. Few students realise how rapidly the number of combinations of factors grows when only two or three variables are combined at a few levels. So, for example, they expect to investigate simultaneously the effect on plant growth of half a dozen soils with as many fertilizer treatments and a like number of lighting regimes. Serial dilution is a mystery as is its mathematical basis - $\frac{1}{10}$ of $\frac{1}{10} = \frac{1}{100}$.

(l) Scaling in all of its aspects is difficult whether it be trying to get some idea of the size of an object under the microscope, the number of yeast cells in a given amount of wine culture, or the number of trees on a hillside. These problems seem so remote and intractable that students see no way of sampling, comparing or estimating. Nor are they able to say that something "seems about right" - is of the right order of magnitude.

The above examples were drawn from the "Energy" classes though most of them we have also met elsewhere.

In the teachers' courses "Size and Scale" and "Light and Color", practically all the mathematical problems encountered were geometrical.

Problems with Spatial Relations

Our work with "Size and Scale" and later with "Light and Color" as topics for teachers' courses focussed attention even more sharply on problems that we had noticed before - difficulties with spatial relationships. Some of these difficulties are so elementary and so basic to understanding shapes and dimensions in the real world that where they exist they could prove serious barriers to real understanding of even elementary aspects of science. As so often happens, these difficulties are hidden; they occur in people who "passed the course" - who were adept enough at manipulating artificial situations to get by with pencil and paper exercises, but who are completely lost when they have to apply these principles in real-world situations.

Our brief was to look for examples of difficulties encountered in learning elementary science. Many of these turned out to be due to difficulties with mathematics, especially with spatial concepts. An outline of the problems of this nature that we encountered is given below; these are factual. Our observations of the causes of these problems is largely speculative and we are not able to support them by experimental data, since we did not have the resources for such detailed investigations, nor were we commissioned to carry them out. We are, however, encouraged to find that our observations are supported by those of our colleagues and by evidence from a number of sources; for example, see Soviet Studies in the Psychology of Learning and Teaching Mathematics, Vol. V.*

While long and painstaking studies are often needed to elucidate these problems, there is plenty of evidence from successful teaching of ways to curcumvent them, and we have included in this paper some suggestions for curriculum developers about some of the failings that we and others perceive in traditional teaching as well as some suggestions for ways in which a more secure grasp of these fundamental concepts, critical to the understanding of most areas of science, may be established.

When difficulties such as these are encountered so frequently and among so many intelligent people, it is hard to avoid the conclusion that the teaching of mathematics in general and geometry in particular is far too abstract and too far removed from the practical situations in elementary science, geography, art, etc. which depend on a sound grasp of spatial relations for their understanding. The shapes and sizes of the real world in their infinite variety become too soon abstracted to small pencilled triangles, rectangles and circles on a small flat piece of paper. Does it help at all to say that the small circle on the page represents a race track or the triangle a roof truss? Our discussions with teachers and college students make it plain that their experiences of geometry were too brief, too stereotyped, and too divorced from real, practical situations.

Geometry as a science has become so formalized that for many pupils it has become the manipulation of small flat shapes on pieces of paper.

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* Soviet Studies in the Psychology of Learning and Teaching Mathematics, Vol. V. Editors Jeremy Kilpatrick and Izaak Wirszup. National Council of Teachers of Mathematics.

It has lost much of its connection with reality and is taught as a purely deductive subject, in which the teaching of abstract principles is followed perhaps by a few practical examples. What is needed is a return to geo-metry, that is to developing and understanding spatial relationships in a real three-dimensional world, and abstracting principles from that. This is a matter not only of reality and relevance, but of scale, of aspect and attitude. The real geometrical objects are seen usually much larger or smaller than the paper abstractions; they are seen from a variety of aspects -- perspective is involved -- and in a variety of attitudes. They are not always horizontal on a flat piece of paper.

GEOMETRY

We chose for our first teachers' course the topic of "Size and Scale" in part because of our own interest in it and in part because of its intrinsic interest and the multitude of fascinating but generally unfamiliar consequences of size in the living and non-living worlds.

Documentation is available of class discussions and teachers' notes, but frequently teachers' difficulties were exposed during practical work in which one or two teachers were working informally with an investigator; in these cases we had to rely on our "mental notebooks" because for one reason or another it was not possible to get a verbatim report. Usually this was because we did not want to do anything to interfere with the atmosphere of friendliness and trust, nor to destroy the spontaneity of the occasion. There is, perhaps, an Uncertainty Principle which applies here, which would suggest that any intrusive sort of recording would disturb the situation being observed -- that is, the teachers' free and spontaneous reactions and comments.

AREA

David Hawkins
p.43-44.

Vol.III,p.48
et seq.

Vol. II.

We knew at the outset that many people are confused about the mathematics of the area of a rectangle. There is here a failure to link "squaring", X^2 and the area of a square. The fact of X^2 as a square built up on the line X is quite surprising to many students. Many students had never even associated the word square in its arithmetical or algebraic context, with its geometrical meaning. Frequently, students add length and breadth instead of multiplying. Confusing terminology creates difficulties, too, as for example between the expressions "ten miles square" and "ten square miles." "...when people get confused that four miles square is four square miles.

Teachers commenting, David explaining

Hedi: That's incredible. It doesn't give you a clue. For miles square or four square miles, I had no conception that that would be different... it sounds like the same."*

Another common confusion is that between area and perimeter. What came as a surprise, however, was the difficulty that the teachers (who have after all, succeeded in the academic obstacle race) had with the basic ideas of area as surface and volume as space occupied. Consequently, our original topic was scarcely touched upon and the greater part of a semester was taken up with establishing a secure grasp of surface and volume and their relationship.

We found a failure to grasp the arithmetical calculation of area as a product of two dimensions; of the two - dimensionality of area. This was also true, incidentally, of the three-dimensionality of volume. So, "I found I had the fromula for area and volume reversed." In the case of circles and spheres, people frequently confuse 2r and r^2 or $4\pi r^2$ and $4/3\pi r^3$, but the terms r, r^2 and r^3, if properly understood as r, r x r and r x r x r should give the clues to one, two and three dimensions and so length, area and volume, respectively. Likewise the need for two coordinates to establish a position on a surface and three for a position in 3D space should give a clue. But these links are lacking.

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* This and other short quotations are taken from the Transcripts and the Teachers' Notes.

A square with twice the length of side has four times, not twice, the area. A similar difficulty arises with the cube law of volumes. We are used to things that increase in linear fashion - two pounds of butter cost twice as much as one; it takes, within reason, twice as long to drive 100 miles as 50, and so on. So we are quite unprepared for those instances of square or cube laws, in area or volume in this instance, which do not conform to the familiar linear progressions. Even when our students had some experience of square law growth, after building up larger and larger squares with wooden cubes, and so had some experience of the effects on area (and volume) of changing linear dimensions. They were puzzled about decreasing the side of a square or cube. So when $2^2 = 2 \times 2 = 4$ is comfortable, $\frac{1}{2}^2 = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ is not. This held even when "half-the-size" was marked on a cube.

This did not mean much

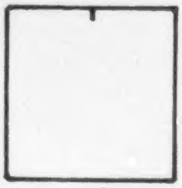

Figure 1

nor did this make the situation clearer,

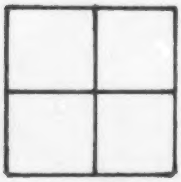

Figure 2

but this did.

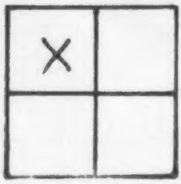

Figure 3

This problem goes deepter than difficulty with area, of course, and lies in the familiar difficulties with fractions and with the notion that multiplying means making something bigger; it goes against our intuitive feeling to multiply and end up with a smaller quantity. Presumably what was happening here was that our students did not visualize the divisions implied, but not concretely shown, in Figure 1, and while they saw them in Figure 2 still were puzzled by $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$, while Figure 3 suggests, however, "There are 4 pieces here so each must by $\frac{1}{4}$ of the original."

Even where these difficulties do not exist, the concept of area as surface is limited and unsure, beyond the idea of L x B - a flat rectangular surface. "...area and surface was interesting. I come up with this - area is the flat space." But little thought is given to the concept of area as surface, as a property of solid bodies in the real world. Another teacher said "I have always thought of area as a flat surface. Never thought of top, bottom and sides." And, "When discussing the area of a piece of paper does one consider both sides?" "To find the area of your foot trace around the bottom of it." BUT, "Surface area includes every bit of skin - top, bottom and sides."

The report* on the Second National Assessment in Mathematics: Area and Volume points out that, "The difficulties shown by students on these exercises seem to result from misconceptions about area rather than computational weakness," which our observations confirm. However, the comment "that an area question refers to the plane-filling aspect of the region" and the sole example of an irregular figure as one made up of two joined rectangles seems to suggest that those involved continue to plough the narrow rut of formalism and that they are little concerned with area as the surface of real objects.

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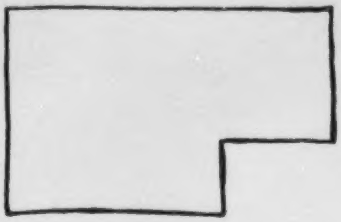
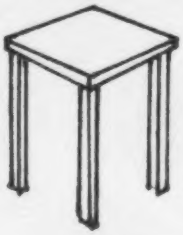
* The Second National Assessment in Mathematics: Area and Volume, by James J. Hirstein. The Mathematics Teacher, Vol.74,no.9, Dec.1981.



Likewise the National Council of Teachers of Mathematics, in their booklet "Measurement,"* say, under the heading "What is Area?" ..."Measuring the area of a plane region is a matter of saying how many of some standard unit of area measure the region contains." Rather disappointingly, Informal Geometry (our italics) deals with area only under "The Plane and its Properties."*

Confusion exists, too, in thinking about the faces of a solid body. Many people speak of a cube as having five sides, and a four-sided pyramid as having four -- ignoring the base in each case, since it is concealed. This way of thinking is revealed when students are asked to draw a network from which the solid figure can be built. Frequently, in the case of a cube, five faces are drawn.



What is the surface of this table?

How many sides has it?

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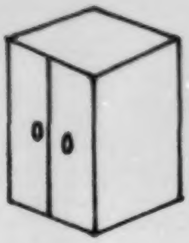
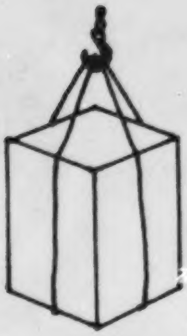
* Topics in Mathematics for Elementary School Teachers, "Measurement", Booklet No.15.

* Topics in Mathematics for Elementary School Teachers, "Informal Geometry", Booklet No.14.



What is the surface of this cupboard?

How many sides has it?



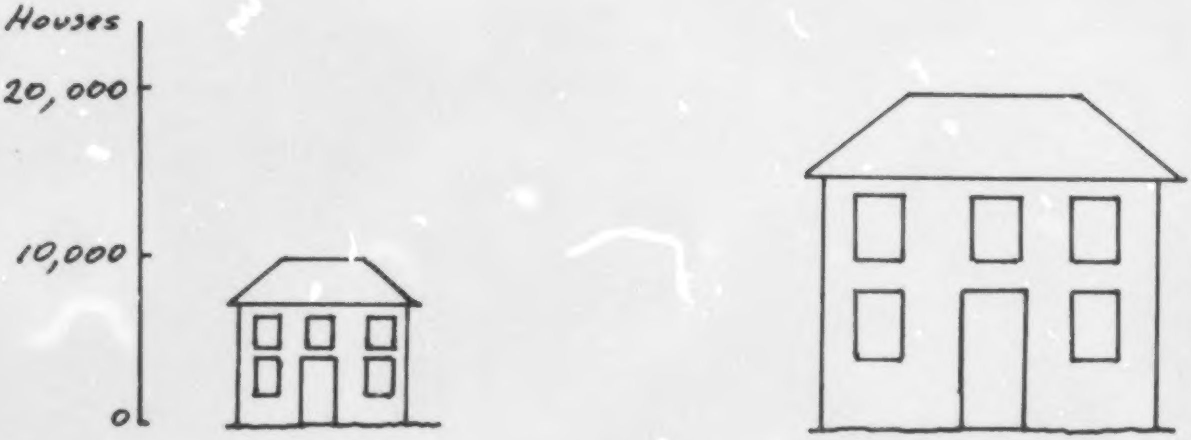
What is the surface of this crate?

How many sides has it?

Although there are three cuboids here (assuming the space occupied by a table to be a cuboid), the perception of surface area is likely to be quite different in each case.

A further confusion is that caused by the imprecision of our everyday expressions of dimensions. Does, "twice as big" mean twice the linear dimensions or twice the area? This led to considerable discussion by teachers, particularly with volumes, because they wanted to construct shapes with twice the volume (or area) as distinct from twice the linear dimensions and so came face to face with cube and square roots. This confusion may cause us to be unintentionally or deliberately misled by graphs which represent linear relations by two-dimensional diagrams.

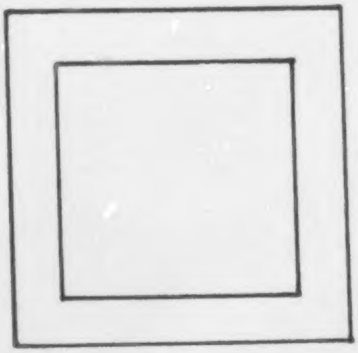
So,



to represent twice as many houses is quite misleading for the unwary.

Compounding these difficulties is the fact that many people have very little practical experience of the units they use in their calculations; this is perhaps an even more prevalent problem in dealing with volume and weight than with area. "I grew up on a farm but I never knew what an acre was till I marched a football field (in a marching band)." This is not at all the same as people not knowing the definition of an acre as 4840 square yards; you may have learned this, but still may have no idea what it looks like, as many people have little idea of a cubic yard. The units themselves create difficulties; square inches, square miles, square centimeters tend to force a focus on squareness which makes it difficult to visualise areas other than those of rectangles in quantitative terms. The acre, on the other hand, does not have this limitation; it may be any shape. A similar difficulty arises with measurement of volume, for example in cubic inches, though in fluid measure we do have volumetric measures that are free from any connection to a particular shape.* This however contributes to other difficulties mentioned below.

Our perceptions can be very misleading. The foundations of a house, out in the open, seem pathetically small and give very little impression of the spaciousness of the completed structure. Distributed areas may seem small compared to compact areas of similar size. So it is surprising that in this diagram the carpet occupies only half the floor.



In these cases and similar ones, one has to train ones intuition at least to the stage of recognizing that these are cases where things are not what they seem. Perhaps we also need to take more account of the way in which we perceive many shapes in everyday life. Heights are often foreshortened, circles such as ponds or flower beds are seen as ellipses, and rectangles such as tables or floors are seen as parrallelograms or trapezoids.

> "A defect in the development of the spatial concepts of secondary-school students is their habit of using stereotypes (stereotyped collections of geometric forms and figures that are studied in class -- the figures are almost always situated in a standard position). There is no variety in the positions selected for the figures; a more diverse collection of surfaces and solids -- such as solids of rotation (the ring and others) -- is not examined.

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* Because, in our measures of capacity, we have volumetric measures which are "dimension-free" it might be as well to teach volume before area, especially as in everyday experience, we deal mostly with solid objects. For people living in a three dimensional world the logical progression 1 → 2 → 3 dimensions may not always be the best.

> The traditional school geometry course does little to promote the development and enrichment of students' spatial imagination. Our experimental work clearly shows the limits (thresholds) of the latter.
>
> Students habitually think of plane geometric figures only in the plane of the drawing, and not in arbitrary positions in space. As a result, they cannot apply the theorems of plane geometry to plane figures in space, especially if the figures do not occupy a standard position (for example, on the faces of a cube).
>
> The teaching of geometry often fails to establish a vivid link between the visual perception of an object's shape and its natural shape. This explains why students sometimes conceive of figures as they see them. For instance, they conceive of right angles as oblique, of an equilateral triangle as scalene, of a sections of a sphere as an ellipse, and so forth. Missing is any work with projective drawings, in which the students would study the properties of geometric figures, using representations, and would solve spatial problems."*

With these difficulties in understanding area-as-surface and lack of familiarity with the size of the units involved, it is not surprising that the areas of irregular or complex shapes -- of a compound leaf or of a radiator, for example -- are looked upon as insoluble mysteries, of little importance or interest and beyond the capacity of ordinary people to calculate, measure, or even estimate. "Why would anyone want to know the area of an apple?" "Are there formulae for volumes/areas of various shapes?" And, rather surprisingly, "How do you find the area of the base of a cylinder or cone without laying it on graph paper?" But after one of our sessions, "I became more aware of the vast area of a blue spruce with all its needles."

How much real meaning, then, do measures have such as bushels per acre, kilowatts per square meter, or compound units such as Langleys and R factors have? Can there be any real feeling for processes dependent on surfaces such as diffusion and absorption or for the morphological developments such as villi, alveoli, or root hairs, which facilitate these processes in living organisms?, for the minute structure of the cell or chloroplast, or even the fins on a heat exchanger? Phenomena such as absorption, evaporation, surface tension, and diffusion, cannot be appreciated without a feeling for the surfaces involved, nor can the power of a concept such as the inverse square law of electromagnetic radiation.

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* Soviet Studies in the Psychology of Learning and Teaching Mathematics. p.55.

If areas as surfaces are not clearly understood, perhaps it is not surprising to find that with cross-sectional areas the difficulty is compounded, for the areas here are hidden. So it is not at all clear that strength of a column, a girder, or a rope is proportional to the cross-sectional area and how its area will vary with the square of the linear dimensions.

Our intuitive ideas of size are usually comparative, so we speak of a big dog or a little horse, knowing that in spite of the adjective the former is smaller than the latter: it is big only in comparison with other members of its class. In making such statements of bigness or smallness we seldom have specific dimensions in mind. When a specific size is used, it is seldom used in terms of the integrative units of area or volume. We use length, breadth and height because they give more information. If we need to cover a 9' x 12' space, then that is the size and shape that is needed, and though we may pay for 12 sq. yds., to specify just that area will not do; it might be the wrong shape. Similarly, though we may buy a 20 cubic foot freezer, we have to know whether it will "go in" - how long it is, how wide and how high. In judging how much it will hold, we are probably guided more by length, breadth and height than by any visualising of a number of cubic feet.

We have, perhaps, a better notion of capacity in terms of pints or gallons, though it is difficult to visualize these in terms of cubic inches; with larger containers, however, dimensions again are our guide.

One does not think of 55 separate gallons in a 55 gallon drum -- it is about 3 feet high by 18" across. And only a ventilating engineer would want to know that a 12 x 10 x 8 room has a volume of 960 cubic feet! We live our everyday lives in a world of linear dimensions; when we depart from these it is mainly to deal with materials which, from their irregularity of shape, will not submit to this mode of measurement: water, flour, wine, etc.

Clearly, the teaching of area in our schools is too limited, too constructed, and too abstract. It is dominated by the rectangle and by the number of unit squares it will contain; it is L x B. Any development of this tends to be to make the computation more difficult, which is likely to confuse the issue rather than to clarify, and which puts even more emphasis on numbers, on abstractions rather than on physical realities. It seems that in school the concept of area is made to fit the units of measurement instead of the reverse. What is needed is an extended and varied concept of area which includes spread, extension, coverage, and surface. A box of dried peas or beans can be spread out. What space does it occupy? What does the space look like when it is manipulated to make a circle, a triangle, a square, a rhombus, a horse? Work with geometrical tiles,* triangles, hexagons, and even circles, will help to give feeling for coverage independent of squares. Peeling an orange, an apple, a banana, a potato extends the idea of surface to solid, non-rectilinear figures. We live, within the ordinary limits of human experiencce on a flat earth and even a round-the-world trip does little to dispel this perception. Early plans and maps -- of classroom, the school, the town -- conform to the idea of flatness and serve well to guide us in direction, distance, and area, but how many people really understand the distortions of area in the familiar Mercator Projection maps? Or the reason why those lines marking shipping and airline routes move in great arcs instead of taking the direct (on flat paper) route? "How is a globe made into a map? I don't know if they calculate areas or what?"

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* See A Pentagon Paper by Barry Kluger and Jane Richtmyer. "Outlook" Magazine, No.14, pp.3-9.

An important aspect of teaching about area and surface in this way is the link with ordinary experience. If area is something that has to do only with flat pieces of paper in the classroom, then the rich experience outside of school will not be used to complement and extend the concept. The many hours, extending over weeks that our teachers spent struggling to master area and volume confirm the woeful gap between formal education and reality in the realm of number and measurement. What does it matter if, initially at least, one cannot attach precise figures? Dressmaking is very much a matter of surface area; so in a different way is wrapping a parcel, covering a pie with pastry, applying fertilizer to a lawn, papering a wall or painting a door.

VOLUME

David Hawkins, p.44-45

Vol.III, pp.100-185

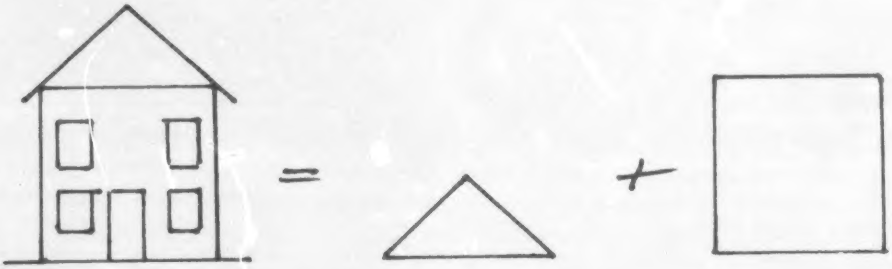
Vol.II

As with the problem of understanding the two-dimensionality of surface, there are problems for some people in grasping the three-dimensionality of volume, and hence in grasping the X^3 in formulae for volume. Linked to this is a failure to see volumes of solids, liquids, and powders as of the same nature - that is, as the "quantity of stuff," or "space occupied." So measures of capacity such as pints, bushels, even cubic centimeters, are regarded as qualitatively distinct from volume measures such as cubic inches. The measurement of rainfall in inches adds another dimension of difficulty, and even those who grasp the nature of this measurement often fail to link this with a volume of water in gallons or a weight in, say, tons.

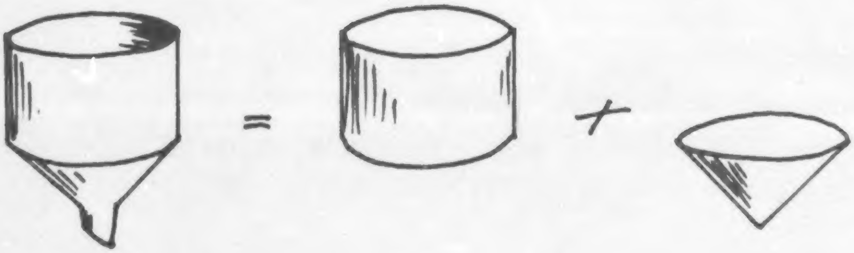
Most people have a reasonable idea of capacity measures such as pints, quarts and gallons from their experience of buying milk, etc. but volumes measured in cubic units are a mystery. "I have a poor conception of what a cubic yard of sand or soil is." This is another example of the unfamiliarity with measures that are known in the abstract, but not in concrete situations. This difficulty extends to the nature of the shapes of solid objects. "I had so much trouble with solid geometry in high school and I don't think I ever handled a cone, sphere, etc. They were always drawings."

The idea of an object displacing an equal volume of water when it is totally submerged should not present difficulties and the fact that changing the shape of the object, for example, a plasticene figure, does not alter the volume is easily demonstrated. However, conservation of volume is not as secure as one might expect, even with such experienced teachers as those with whom we worked. "Does the volume of any object change when the object, a ball or a submarine, sinks below the surface? Maybe it doesn't displace more water." There are problems, too, with pore space. We may speak of a cubic foot of soil, easily measured, and still add a quantity of water without changing the overall volume. Similarly with the question, "What is the volume of a sponge?"

Some measure of the uncertainty that can exist is illustrated by the questions, "You do measure volume in square feet, don't you?" (in connection with the volume of rock eroded from a canyon), and "Can you have a cubic gram" (in connection with the volume of water used in taking a bath). Several students, given a practical problem of finding out how much hot water they used could not find the volume of a bathtub "because it isn't a rectangle" even though the average bathtub is a very good approximation of a cuboid. Just as students are afraid to approximate or use ball park figures, so they do not approximate figures, either two dimensional or solid. The process is rather more complicated here because, though it may involve squaring off or paring down, it may also involve analysis into component manageable shapes and the addition of these:



or its 3D analogue



The fact that a cubic centimeter of water weighs a gram, so convenient when one has become familiar with it, is a frequent cause of confusion to the uninitiated.

As with area, there is great difficulty in understanding the effects of increasing the size of a figure. It comes as a very great surprise that doubling the linear dimensions increases the volume eight times (2 x 2 x 2).

"We were surprised that a 2" pyramid took one container (of salt) and a 4" eight times as much." This was after work using wooden blocks to build up cubes of various sizes to establish the cube law of growth. Even when the cube law has been established with cubes, there is no recognition that all solids will grow similarly by a cube law. Perhaps a basic problem here is that in most everyday relationships we deal with linear growth rates, and thus non-linear relationships are unfamiliar and difficult. "I wonder why I have so little visual intuitive understanding of the growth of area and volume?"

Imprecise language is sometimes a difficulty, and "twice the linear dimensions" or "twice the length of side" were often equated with "twice as big." "One of the problems with area, volume, and mass is to come up with a precise definition." Our students repeatedly asked about constructing a figure "twice the area" or "twice the volume" of a given figure: they were in fact ready to get involved with square and cube roots but were not expecting anything so complex.

N 1. Lobachevskii, in his textbook of geometry, wrote about the two relationships in which we find ourselves in relation to space - that in which we are surrounded by space and must find our position in it and that in which we are external to the object occupying space and look upon it -- "myself within" and "myself looking upon." Our observations of teachers' difficulties with area and volume and other problems such as projections mentioned later suggest that formal math and science teaching frequently fails to give students a sufficient fluency in dealing with two and three dimensions, either of "space occupied" or "position in space."

In this latter connection, the construction of graphs does not, as is sometimes assumed, always establish the notion of Cartesian Coordinates as determining position. Students seem to think only of number or quantity in this connection: the link with latitude and longitude, or map grid references, or even games such as Battleships, is not made.

VOLUME/WEIGHT RELATIONS

Perhaps rather surprisingly, the constancy of the volume/weight relationship in uniform materials was not always realized. It was not absolutely clear that if one puts twice the volume of, say, clay on a balance it will weigh twice as much. "I'm not clear at all on the relationship of weight, volume and surface." "What is volume? It is the amount of space occupied. What is mass? It is the absolute determining quantity of matter. I'm not sure that definition makes sense to me." Weight and volume sometimes seem to be the same, but not necessarily.

> "David: You mean the same stuff? You mean that if you are only dealing with water or only with potatoes? Then twice the one, is twice the other? But I agree, it is very easy to ...
> Sally: Twice as much water wouldn't weigh the same as twice as much potatoes?
> Voice: If you increase the volume of water by twice, then the weight would increase twice as much, too?"

With these uncertainties it is not surprising that the concept of density is difficult for so many people and that dependent notions of floating and sinking, or convection are even more so. And if density is such a vague notion, relative density adds a further degree of confusion. "Relative density means that you must always be comparing something. I never knew that." Some of our everyday descriptive terms do not help. Cream floats on milk and so must be lighter, yet we speak of "heavy cream" and of "heavy oil" and other substances where "heavy" and "light" have to do with consistency and not weight.

The difficulty with the volume/weight relationship reappears when it is suggested that with a material of more or less uniform thickness area is proportioned to weight, so that one can find areas by weighing.

"It was fun to find out that you could find the area (of an orange) by weighing the peeling." Even then some did not at first understand why it was necessary to compare the weight of the whole peel with that of a unit area of peel.

SURFACE/ VOLUME RELATIONS

The unfamiliarity, in everyday situations, of thinking of area as surface and therefore as the property of an object (be it as large as the earth or as small as a pin) means that the very idea of a surface-to-volume ratio comes as something quite new. That this ratio should change with altered dimensions even though the shape remains constant is a great puzzle. "As the volume increases the surface-to-volume decreases. I don't know why this should be clearer (after practical measurements) but it is."

The effect of shape on surface-to-volume relations is easy enough for people to understand once it is pointed out, but the idea doesn't seem to have occurred to most people until their attention is directed to it. Then examples come quickly enough. "Oh, when you take a ball of dough and roll it out into a tortilla": "Stretching a clay cube into a different shape doesn't affect the water displaced. I guess the volume remains the same but the surface area changes," and, reaching towards a more sophisticated concept, "Is the reason that a large drop of water is less round because its surface/volume is less?" The connection between critical barriers and the history of science, dealt with elsewhere (see Abe Flexor) seems to apply here; the consequences of varying surface-to-volume ratios received little attention from scientists until relatively recently when biologists such as D'Arcy Thompson and Julian Huxley wrote about it.* In elementary science and mathematics it is rarely touched upon.

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* D'Arcy Wentworth Thompson, "On Growth and Form", Cambridge University Press.

* Julian Huxley,"The Size of Living Things", Outlook, No.13, Autumn 1974.

There seems to be a duality in the problem associated with understanding how surface-to-volume ratios vary with increasing size. First there is the unfamiliarity with the amount by which area and volume increase with linear dimensions - that is, the surprisingly rapid growth, particularly of volume as a physical quality: twice the length, eight times the "size" and, allied to this, the non-linear growth of the numbers that appear as measurements are made and calculations carried out. Second, there are different rates of change -- linear, square law and cube law -- in growth of edges, surfaces and volumes with the changing ratio between them. The properties of similar geometrical figures are looked on as invariant, all cubes have so many faces, edges, right angles, so how can any relations between these change?

> "Today we worked with volume and area.
> How many faces are on a cube?
> We looked at 1 cube.
> Volume 1 - 6 area.
> What is the next larger cube?

	1	2	3	4	5	6
Vol.	1^2x1	2^2x 2	3^2x3	4^2x4	5^2x5	6^2x6
Vol.	1	8	27	64	125	216
Area	6	24	54	96	150	216
				(16x6)	(25x6)	(36x6)
	5	16	27	32	25	0

Difference between area and vol.

	7^2x7	8^2x8	9^2x9
Vol.	343	512	729
Area	294	384	486
	-49	-128	-243

> "Marsha and I looked for a set number relationship throughout. I found that the same number relationship does not exist with next larger cube.
>
> Marsha and I tried to find some set ratio between the difference of vol. and area for each cube.
>
> The area starts out being much greater than volume."

In investigating these relationships, numbers can cloud the visualization of the physical properties, so "Area starts out being much greater than volume," "As volume gets bigger area gets smaller." However, it is not difficult to devise simple experiments to show the importance of the ratio of surface to volume in practical situations; for example, "Cooling water in various shaped containers and relating it to surface/volume was stimulating."

Since this ratio is of such enormous consequence in explaining so many aspects of the form and function of living organisms, and so many physical phenomena -- the shape of water drops, the rise of liquid in capillary tubes, cooking times, ice melting times -- failure to grasp it may indeed be a critical barrier to one of the great integrating themes of science.

Problems with surface and volume occur in everyday situations. A sheet of paper is thought of as "all surface and no volume" -- it has length and breadth but no obvious thickness; one cannot see an edge to be measured. Students are frequently baffled when asked to measure the thickness of a sheet of paper, even though the thickness and volume of a ream of paper or a book are obvious. A gallon of paint has obvious bulk, but when it is applied it seems to be "all surface." "How many cubic yards of gravel are needed to cover the driveway with a layer three inches thick?" is a similar problem that many people find difficult to tackle, short of buying the gravel and seeing how far it spreads. This problem is similar to those with the ball of dough and tortilla as well as to the situations mentioned above. If you take a bulk of material and spread it out, what will it look like? How far will it spread? If it is to be of a particular thickness, what area will it cover? At least one of our teachers mentioned having encountered this situation: What would a "yard" of gravel look like? How big a job would it be to spread it? What area would it cover to the required depth? How thin, then, can a layer of matter be made? What is the practical limit? The familiar experiment of putting a drop of oil of known volume on water, allowing it to spread and measuring the area gives a surface of large area, but one that is only about one molecule thick. At a more elementary level, to fill volumetric measures with rice or beans and then spread them out in an even layer gives a feeling for amount of material in its bulk form and as a surface cover.

Problems with learning about volume are somewhat similar to those with area in that the basic concept, in this case space occupied, is not clearly understood. When it is dealt with, it tends to be only within a realm of shapes dictated by the units used scuh as cubic inches, so that volume in these terms is limited to rigid bodies which are roughly cuboids. But in the case of volume there is the complicating factor of a totally different set of measures for substances whose shape is that of the container, be they pints of milk, cups of sugar, or gallons of paint. To make things more confusing our measures -- the cups and gallons -- are almost invariably roughly cylindrical. More illogical yet, when we deal with even more tenuous substances such as natural gas we revert to cubic measure!

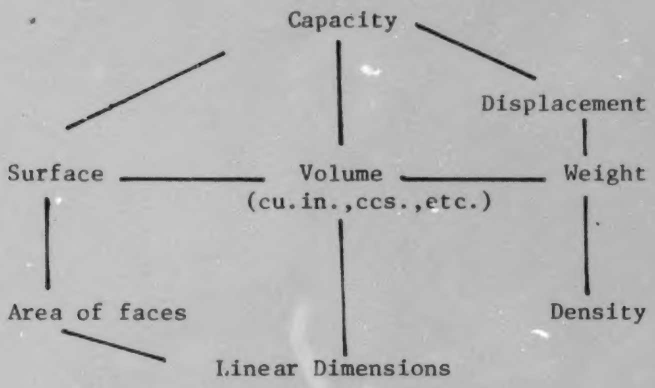
A great deal more practical experience, at all levels of schooling, is needed by most people:

(a) in observing and manipulating objects and materials to get a clearer idea of "space occupied."

(b) in interconverting measurements in terms of capacity and cubic measure, for example, constructing cardboard cubes of various sizes and filling them with sand or rice and comparing them with cups, pints, etc.

(c) in establishing the proportionality between space occupied and weight or mass with any given material.

(d) in becoming familiar with the varying weights of constant volumes of dissimilar materials and conversely the varying volumes of constant weights, so that the idea of density as "amount of stuff" is clearly established.



Ideally, an intuitive network of ideas such as that indicated in the above diagram would be developed concurrently and not sequentially as isolated topics. More formal understanding of each topic could develop later. As it is, the dictum of "teach one thing at a time" is so dominant that in attempting to remove the difficulties of complexity we create new sets of problems.

Would our teachers have reached a better understanding if early in their schooling they had been given experiences such at these:

-- Take a brick (ideally one without holes or keys such as a "paver");

-- Measure length, breadth, height;

-- Wrap it in paper to make a parcel. Use the smallest piece of paper to do the job properly;

-- There will be some overlap of paper. Try to cut out overlapping pieces so that there is a single layer;

-- Cover all sides with 1" (cm) tiles. How many are needed?

-- Cut out pieces of paper or card for each side. Assemble them to cover the brick. Compare with the original cover.

-- Find out how many ways these can be joined into a single piece and still cover the brick.

-- Make a box with a lid from these pieces. How much sand does it hold in cu.in. (cubic centimeters), pints, etc.?

-- How many 1" (1 cm) cubes to cover one of the paper faces?

-- How many layers to build a full-scale model brick?

-- Repeat the above starting with different faces.

-- What is the total number of cubes used in each case?

-- How much does the brick weigh?

-- How much water is displaced when it is put in water? (A displacement can be made from a bucket with a piece of pipe inserted in the side)

-- How many pints, etc. of water are displaced?

-- How many cubic inches?

-- How much heavier is the brick than the water displaced?

-- How many times heavier?

-- How many cubic inches, pints, of water weighs as much as the brick?

-- How does this compare with the volume of the brick?

-- How much does the block of wood (identical in dimensions to the brick) weigh?

-- How deep does it float?

-- How much water does it displace?

-- How much of its volume is below the surface?

-- How does the volume and weight of the displaced water compare with that of the wood?

-- How heavy "a cargo" can the wood carry and still remain afloat? What happens as it is loaded? How much water is displaced? What does it weigh?

-- How does this compare with the combined weight of ship and cargo?

All this should be carried out with plenty of time to watch, to puzzle, to reflect, to experiment, to compare.

Since area and volume were the topics of principle concern in the Size and Scale course, there is adequate documentation of them in Vols. II and III. In the Light and Color course the mathematical difficulties that arose were incidental to the main purpose of the course, the understanding of light. So the documentation does not adequately record the nature of these difficulties, which usually arose when the class had split up into small groups working independently, with the investigators circulating among them. The record which follows is based largely on personal recollections of the investigators.

PROJECTIONS AND SHADOWS

David Hawkins, p.24-34

Shadows, like many other things around us, are often seen but seldom observed. They are rerely deep enough to obscure totally the surface they cover, so one tends to see through them to the surface itself. Shadows as lack-of-light, and the geometry of their appearance as obstructions of a beam of light, came as a surprise, even to people conscious in a vague way that they lay in a direction opposite the source of light. This matter arose first when three projectors with colored filters were being used to observe additive color mixing. A number of colored shadows were produced. But shadows are black; how can they be colored? Even when the situation was simplified with only red and white light used, so that red and black shadows were cast, there was the greatest difficulty in analysing the situation to see which beam of light was responsible for which shadow.

Even a black and white slide projected in the familiar way was not recognized as a mosaic of transparent areas allowing light to pass and darker areas obstructing the passage of light to a greater or lesser extent, and so casting shadows.

The fundamental scientific problem in all this is the failure to think of light as traveling, being projected outwards from its source like a high powered jet of water or a stream of machine gun bullets. If one were in the path of water or bullets it would be possible to seek shelter behind some solid object; one would be in its shadow. Only when light and shadow were explained in such concrete terms were all the teachers happy with the idea of shadow as the absence of light - of light rays being projected outward in straight lines and having their path obstructed. Once that realization was achieved, there still remained the geometrical difficulties, first of all, the parallel nature of the sun's rays and second, the divergent rays of a projector. Students found it difficult to visualise the light from a slide projector forming a cone or a long narrow pyramid. That the image is inherent throughout the length of this was also not realised, the common conception being that it somehow materialized on a screen put in the path of light.

Teachers also found it difficult to predict the effect on the shape of shadows when the screen was oblique to the projector, though this occurs often enough in the real world -- shadows in sunlight are almost always oblique. Hence, the difficulty people have in producing a round shadow from say a handball, when asked to do so in sunlight. Only after a period of trial do most people realize that they need a surface normal to the path of sunlight.

Difficulty is encountered in visualizing the shadow that will be cast by a solid object, even a simple shape. So, when asked to make a rectangular shadow in sunlight with a coffee can, one teacher said "You can't make a square shadow with a round object." This belief is quite common.

String models do not seem to help very much. When a demonstration was set up to predict the shape of a shadow using string to suggest the path of light rays from the projector, past the edge of the shadow-casting obstruction, onto the screen, the teachers were not at all convinced of the connection between this and the shadow that would be cast when the light was switched on, since they still had not thought of light travelling.

The most important consequence of these difficulties may be that some people do not understand cross sections which are used so freely in our teaching. If you "can't make a square shadow with a round coffee can", what meaning does a vertical section of the can, shown as a rectangle, have? More is said about this later.

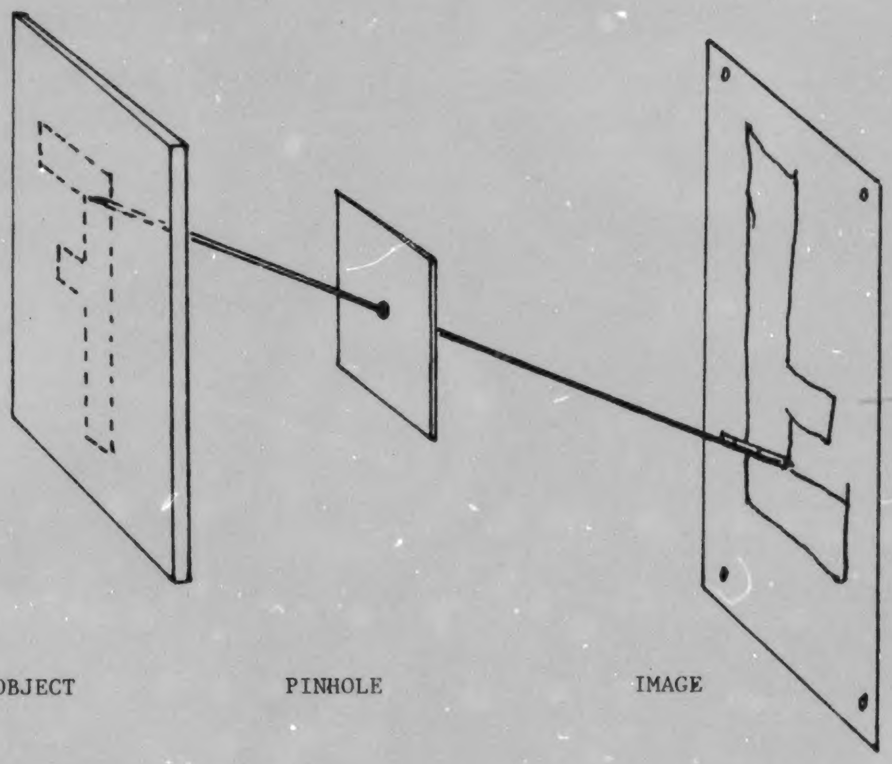
Projection, in the sense of extension of an existing line or plane, albeit an imaginery one, caused difficulty, too; though the concept of the equator is familiar enough and well understood, for example, the idea of its projection outwards to form the celestial equator troubled students.

Everyone is familiar enough with the idea of an angle, yet working with mirrors, we found that concept to be somewhat limited in some students. An angle is something you look into; it is a corner. So students were quick to grasp the "kaleidoscope principal" when we were working with hinged mirrors, and to use these mirrors to find the connection between the number of images and the angle between the mirrors. When, however, they had to use angles as projections outwards they were in difficulty - in predicting where a ray of light would have to strike a mirror to be reflected in a certain direction, in thinking about perspective, and above all in estimating where images would appear in a mirror. But in the latter case, while they could not estimate equal angles of incidence and reflection, there was no difficulty when the problem was transposed to a drawing of a pool table, with the problem of "putting the ball off the cushion." Yet it is just the concept of angles in connection with projections that is so important in surveying, astronomical measurement, photography, geometrical optics, etc.

The concept of angles with the observer at the apex, with the angle projecting outwards, of angles as a divergence from a point, was not well grasped, nor was that of an angle as a turning about an axis. If this is the case, then the discomfort over angles greater than 180° becomes more understandable. The difficulty of visualizing angles in space, in this case the direction of light beams from two projectors, seemed to be the underlying difficulty in analyzing the situation of the colored shadows. The explanation of the umbra and penumbra raises similar difficulties.

The apparatus, shown on the following page, was used to help students visualize the path of light rays through a pinhole, but even then they were not convinced when a rod was used to simulate the path of light rays. Only when holes were punched in the "object" card outlining the "F" so that a light could be shone through them and marked on the "image" card did the geometry become clear.

Two questions which arose were: "Why was the F enlarged?" and "Why was the F upside down", which suggests that not only are abstract angles not understood, but that even in concrete systems such as levers, balances, auxanometers and instrument pointers the angles are not understood. This means that the idea of similar triangles so much used in measurement, and the three basic trigonometrical ratios, is uncertain in such dynamic situations.



SIMILARITY

In the standard geometry course a great deal of attention is given to congruent triangles and relatively little to similarity, yet it is the latter concept that is the more useful in its application. The concept of similarity is difficult to grasp. Ideas of identity, and of equality seem to be quite straightforward to most people; ideas of proportionality, in contrast, are very difficult.

The idea of similar triangles crops up in such situations as height-finding by the shadow method, optical levers such as the reflecting galvanometer, and levers of the second and third orders, in which there is a common angle between superimposed triangles, range finding, trigonometry, etc. The idea of similar triangles crops up more frequently when the similarity depends on vertically opposite angles, as for example, in levers of the first order, in estimating the diameter of the sun from the penumbra, and similar situations, particularly those in geometrical optics involving such phenomena as the pinhole. Here, point-to-point similarity based on triangles leads on to the wider case of similarity of other shapes in object and image, to the reversal of images, to magnification or diminution of images, to projection, etc. These are static situations. Dynamic situations involving similar triangles arise, as mentioned above, with equal and unequal arm balances, and with magnifying pointers such as instrument pointers and auxanometers. Here we have triangles in which the third side is imaginery.

While similarity has a wide range of application in the case of similar triangles, the more general case of similarity as sameness of shape across the whole gamut of shapes, both plane and solid, needs to be developed, perhaps in connection with the idea of magnification. The enlarging (or diminishing) lens changes apparent size but leaves the shape unchanged; the image and object are similar figures. Perhaps ideas of similarity in plane figures might be developed by projecting slides of them, and tracing round the images formed at various distances. If this were done on paper the figures could be superimposed so that identity of angles and proportionality of sides could be established practically.

SYMMETRY

The problem that some students encountered with the magnification and inversion of the letter "F" when traced by rods through a "pinhole" indicated a lack of feeling for symmetry or balance. This is an idea inherent in most work with geometrical optics -- that a straight rod balanced on a fulcrum, in this case the pinhole, will trace out, as it is moved, precisely the same shape on the other side as on the side that is moved. This image may be larger or smaller in the ratio of the two arms; it will be magnified or reduced.

PERSPECTIVE

Everybody knows that distant things appear smaller. But students did not use this knowledge when trying to understand the mirror image of the room in which they were working. They could not perceive the depth of the image, since they appeared to see distant objects as they knew them to be and not as they really appeared. Only when it was pointed out that the actual image was a great deal smaller could they relate the diminished image to perspective, that is, to distance, and so accept the depth of the mirror image. Teachers have told us of instances where young children watching a playmate walk away saw her get smaller and again get bigger on the return and asked if she "felt different." While adults would not make that mistake, they are often uncertain about some aspects of perspective. They do, however, make the opposite error when their experience of the real size of objects overrides their perception of the apparent size. Angular distances and the artist's method of measuring the apparent sizes of distant objects are unfamiliar. Parallax is little understood.

Here again we meet the problem that in the real world we seldom see shapes as they are; tall objects are foreshortened, circles are seen as ellipses, rectangles as parallograms or trapezoids, and so on. We do not see the faces of a cube as six squares but, more probably, as three rhombuses.

SECTIONS

The difficulties encountered with shadows and projections suggest another category of problems which we have not specifically investigated but which do cause a great many problems, that of diagrams, and particularly sections. Almost every one of a large class of students in the Energy course objected to the suggestion that the amount of sunlight striking the earth would be proportional to the cross-section of the earth and not of the hemisphere facing the sun. This was not another case of the "can't do's" being in difficulty again for even the more mathematically competent students were vocificerous in their disagreement. This appears to be a difficulty in the shadow/projection/section area. Most science and math teachers rely heavily on diagrams to explain their work, but these are often not understood and are frequently misleading; they probably fall into the category of barriers which we have labelled as pedagogenic - those which are the result of teaching, not necessarily of poor teaching, but teaching that is unaware, lacking in a conscious effort to understand and forestall the difficulties of the learner.

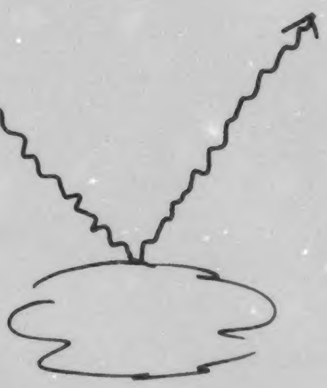
Sometimes a concept that is grasped verbally is confused by a diagram. This may occur because the diagram adds extraneous detail that clouds the issue. But sections, which are used so freely, create difficulties of their own. Geographers have difficulty in getting students to read contours; they may be able to define what a contour is, and even follow the procedure for drawing a map section, without having any clear vision of the real-world, three-dimensional shapes involved. Biologists complain that their students cannot build up a three-dimensional picture from serial sections. How many people can say what shape is obtained by cutting a corner off a cube or a slice off a sphere? How much confusion, then, is generated when we draw a square section of a round coffee can for people who believe that you can't make a square shadow with a round can? And how meaningful are calculations about relative strengths of beams,dependent on cross sectional areas, when ideas of sections are so vague.

DIAGRAMS

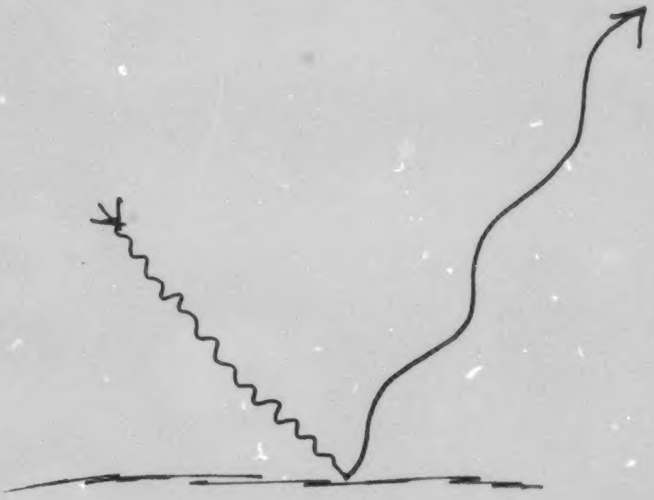
"PEDAGOGENIC" PROBLEMS

Difficulties that are caused by the use of sections are one class of a larger group of difficulties caused by diagrams, a group which itself is part of the category of difficulties which we have labelled "pedagogenic." Of these a common one is the representation of AC current or electromagnetic waves as sine waves when it is not made absolutely clear to students that these are graphical representations of variations in something which we may call amplitude, plotted against time, and not physical shapes. The word "oscillation" does not convey this wrong impression, but a diagram does.

Diagrams similar to this:



REFLECTION



RE-RADIATION

used in connection with the earth's heat balance and the greenhouse effect can create misunderstandings which verbal descriptions do not. "Short wave solar radiation is absorbed by the surface of the earth and reradiated as long wave radiation" is pretty unambiguous. The diagram, presumably, is responsible for statements such as "short wave radiation is reflected back as long waves."

Herbert Lin* discusses difficulties caused by drawings, for example:

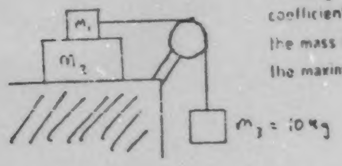
"5.2.1 - Literal Interpretation of Schematic Drawing

A picture provides a schematic representation of a physical situation - it offers qualitative information about the spatial relationships in the problem. Therefore, quantitative irregularities in the picture itself are assumed to be insignificant in the same way that a stick figure merely represents a person, and is not meant to depict a person who really looks like the stick figure.

This type of abstraction causes difficulties for some students, who will misinterpret certain perceptual aspects of the picture (often hand drawn). To the picture's artist, these perceptual aspects are often irrelevant (e.g., he does not care about the length of the side of the cube he has drawn, or that the block is not perfectly square.). However, the perceptual image is so compelling that the student may mistakenly ascribe physical significance to these aspects.

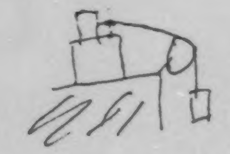
Example 5.2.1.a

A student came in for help on the following problem:



Mass $m_2 = 10$ kg slides on a smooth table. The coefficients of static and kinetic friction between m_2 and the mass $m_1 = 5$ kg are $\mu_s = 0.6$ and $\mu_k = 0.4$. What is the maximum acceleration of m_1?

I drew by hand the picture below, and asked him to describe what would happen if he had the situation posed by the problem:



S: I'm confused... you can't do it.

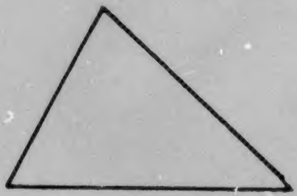
H: Why not?

S: Because the blocks will fall over."

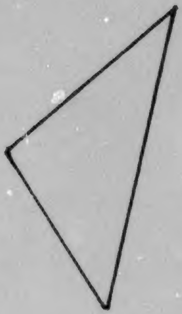
---

* Herbert Lin, Problem Solving in Introductory Physics: Demons and Difficulties, Research Paper, MIT, 1979.

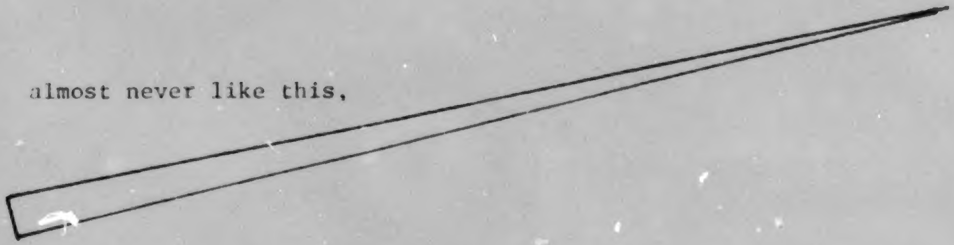
The stereotyped diagrams used in formal geometry lead to very limited conceptions of some figures and relationships. For example triangles are almost always shown like this,



hardly ever like this,



almost never like this,



though this latter is the sort of shape that occurs frequently in real life situations such as surveying and astronomical measurements.

LINKS There are connections, which the diagrams below attempt to indicate, between various of these geometrical topics. Many are obvious, but some, such as the suggested links between ideas of projection and shadows and between these and sections, have not been sufficiently investigated, if indeed they have been studied at all. But these links are more likely to become apparent in practical situations than in formal studies, where usually we are trying to abstract principles from the complexity of the real world rather than using that complexity to link concepts. So, in using a slide projector to project an image, if one is interested in the process and not only the image itself, one is involved in ideas of projection -- of light travelling in straight lines, of shadows as the absence of light; of the geometry of the beam of light as a long narrow cone or pyramid; of the divergent angle of the beam; of the square law involved in the increasing area of the image as the projector is moved back and the inverse square law as the same amount of light covers an ever larger area; and of the idea of sections if the beam is intercepted at various angles by a sheet or card.

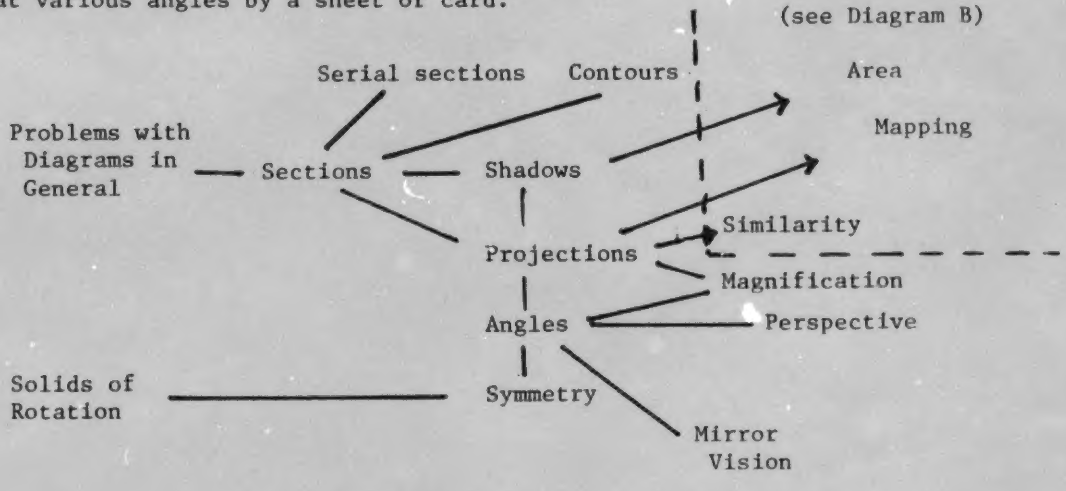

(see Diagram B)
Serial sections
Contours
Area
Mapping
Problems with Diagrams in General
Sections
Shadows
Similarity
Projections
Magnification
Angles
Perspective
Solids of Rotation
Symmetry
Mirror Vision

DIAGRAM A

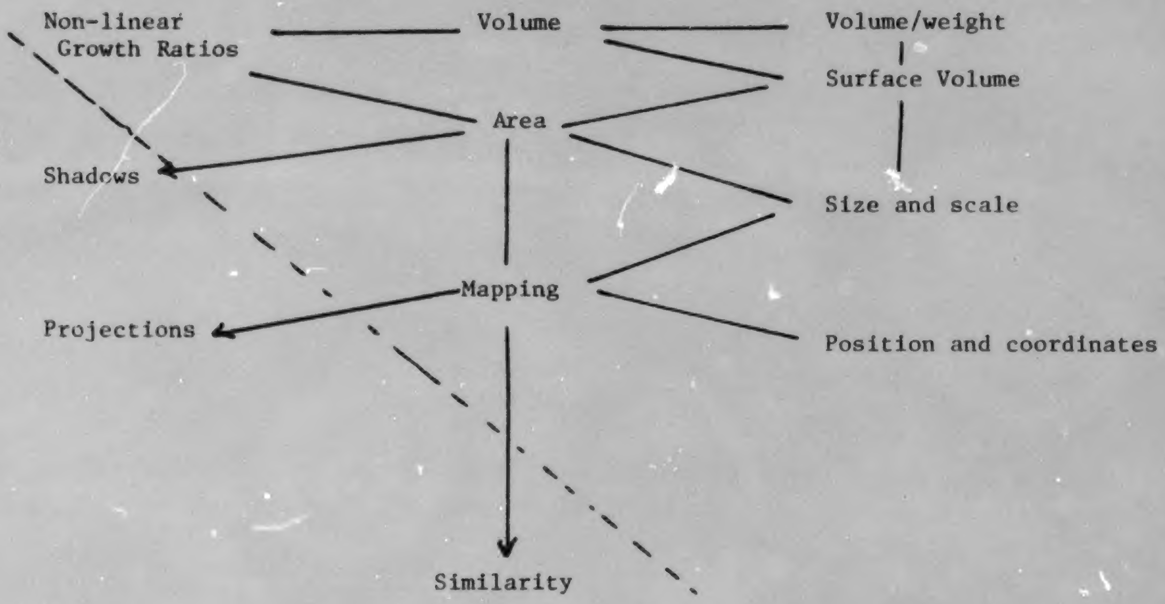

Non-linear Growth Ratios
Volume
Volume/weight
Surface Volume
Area
Shadows
Size and scale
Mapping
Projections
Position and coordinates
Similarity

DIAGRAM B

One wonders how many teachers using a slide projector think of this group of connected ideas, either in connection with the light itself or the spatial relations involved. But they have all presumably learned about such things in their mathematics education. The question is whether in a practical teaching situation one might not have a better grasp of these concepts and their relationships if some of the geometry were learned from the real world situation, instead of being learned as a set of abstractions to be applied (perhaps) to the real situation. This suggests two radical departures from established practice: 1) to move from the complex to the simple, and 2) to teach in an "anastomotic"* rather than a linear style.

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* From the branching network of conducting elements often found in living tissues, especially higher plants, which provides a redundancy of connections so that if one channel is interrupted, alternate pathways are available. In an educational sense this means that if important concepts are approached through a variety of examples and experience, a "blockage" in one of these will not inhibit subsequent stages of learning and furthermore, the knowledge will be more securely established having been reinforced by the multiple access.

As another example of this style of teaching, take the simple measurement of shadow length of a vertical stick in sunshine. Over time this will tell one a great deal about the apparent movement of the sun, of the direction of its rising and setting, of due south and solar noon, of latitude and longitude, and of the seasonal changes in solar attitude. By comparing the relationships between the height of various objects and the shadow lengths at a given time, one finds that one has a series of similar triangles; by simple scale drawing it is possible to find the unknown height of an inaccessible object from the known height of an accessible object. Further, if the exercise is carried out with a number of sticks of known height and the ratios of their height to the shadow length at a given time is calculated, the results are constant, giving the tangent of the angle of elevation of the sun. By repeating the exercise at various times of day it is easy to produce a simple tangent table, one that means something even to older primary school students. One group of teachers set a series of one to ten centimeters Cuisenaire rods vertically along the edge of a piece of graph paper and marked the shadow lengths. They then had a series of similar triangles with perpendiculars 1 --------- 10 in which the constant proportionality between the sides could be shown, and from which simple trigonometrical tables could be constructed by by repeating the observations at various times of the day. Incidentally, the triangles, when superimposed, produced a convincing diagram of the parallel nature of the sun's rays.

CONCLUSIONS

The mathematical difficulties met with in the elementary science with which we dealt were threefold:

(a) Lack of fluency in performing simple computations. This was not only a matter of multiplying 235 by 3, say, and making a mistake, but of not seeing almost immediately that the answer is about 700. This lack of fluency not only impedes students in their own calculations but makes it difficult for them to follow mathematical arguments developed on the blackboard.

Coupled with this, and even more serious, was the frequent difficulty of knowing "whether to divide or multiply", that is of seeing the logic behind the figures.

(b) Failure to be able to think numerically and to value quantitative statements as a more precise amplification of non-quantitative language: "A is bigger than B" tells so much; "A is twice as long, or four times as heavy or has three times the area of B" adds much precision to the description. Furthermore, the use of simple arithmetic in testing statements was generally lacking. Is the statement, "A is bigger than B" really true? If so, how much and in what terms -- weight, area, volume, length, etc. Is the difference significant? Is it worth bothering about in the circumstances and on the scale with which we are concerned?

These two difficulties were encountered mainly in the Energy courses, where a good deal of simple calculation was required and where the sense of many arguments depended on a quantitative assessment of the situation.

(c) The third difficulty, arising in all courses but particularly in "Size and Scale" and "Light and Color," lay closer to the heart of our interest in critical barriers because it was not concerned only with the manipulation of numbers but with our students' intuitive understanding of the nature of the space around them and the scale, position and movement of objects in it. One can have a nodding acquaintance with much elementary science in qualitative terms without becoming involved in calculations, but so many concepts in physical science depend on having sound ideas of volume, angles, surface, etc. that an unsound grasp of spatial relations can result in confusion over the fundamental scientific ideas themselves.

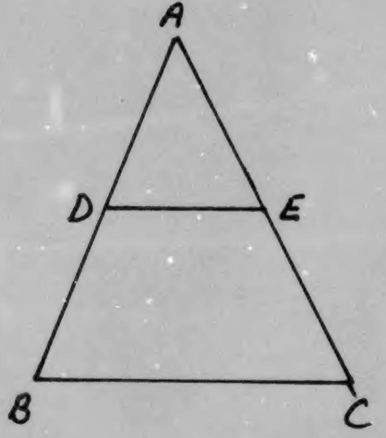
There seems little doubt that our students had received an education in geometry that was too hurried, too removed from practical situations, too abstract, and with too rapid generalization from too narrow an experiential base.

Geometry originated as a practical technique for measuring land. For many practical people, it remains, with its sister discipline of trigonometry, an essential tool for dealing with daily problems met in work or leisure. The builder constantly uses his level or plumb line to establish horizontal and vertical planes and his 3-4-5 triangle to square the foundations of his buildings. The carpenter uses his squares and meter gauges for right angles and 45° angles and other simple methods to establish the angles of roof beams; the gardener produces circular or elliptical beds with string and pegs; the sportsman estimates directions of flight and of a ball; the homeowner tessillates with floor tiles, plumbs the line of wallpaper, estimates areas of paper or curtain material, positions furniture and fittings, perhaps orients a solar collector to face due south; to have an elevation and an angle close to that of his latitude, and establishes the elevation and azimuth of the sun. All these people are concerned with spatial relations and are using practical geometry in one way or another, often with scant understanding of textbook geometry. No doubt their practical experience could easily be brought to bear to help them to assimilate formal textbook geometry. The reverse by no means follows, and as our examples show, those who have taken courses in school geometry, and who may even teach this subject, may be left floundering when called upon to use simple geometrical principles in practical situations.

The "square eye" that the skilled craftsman brings to his work and the skills of judgment and estimation are part of the trained intuition that he is able to apply to his work (see Herbert Lin op.cit.); these skills are not innate but become ingrained. While one would not expect our schools to train their students to this level of skill, one would hope that they might provide sufficient practical experience with spatial relations so that intuitive understanding was brought to a higher level of precision.

Informal, out of school experience is continuously assimilated into the developing intuition; the formal training, if not somehow linked to this experience, is likely to be not always easily accessible. It is as though the learner had always to hunt through the tool box for a square or a level, without having the confidence or skill to rely on judgment. At best this makes the process clumsy and uncertain; at worst the tools are missing.

Other researchers have pointed out the difficulty that some students have in connecting reality and representation. This is also a lack of transfer between what has been taught formally and situations which we teachers see as comparable but which may be vastly different from the students' viewpoint. For example, the theorem about similar triangles is usually built around a diagram such as this:



of this size and with this orientation. Perhaps it is not surprising if, given this situation:

with an 80-foot tree and a 6-foot pole, students fail to see two similar triangles where the only fairly obvious angles are the right angles, there are no visible triangles, the imaginary triangles are differently disposed, the orientation is vertical, and the scale is vastly different. Certainly memorizing a formal proof does not by itself prepare one to us it, or recognize a parallel situation, in a totally different context. If, however, the principle has been met in a variety of contexts and the generalization made from them, it is much more likely to be recognized in another new situation.

Wirsup* quotes Van Hiele* on the question of building up the conceptual framework from which the learner may move out to tackle new situations:

> "After analyzing a typical lesson in a geometry class, P.M. van Hiele (1959) writes:
>
> The teacher reasons by means of a network of relations which he comprehends, but his students do not. On the basis of this network he presents the mathematical relations which the students end up manipulating out of habit. Or, rather, the student learns to apply -- out of habit -- these relations of whose source he is unaware and which he has never seen.
>
> Apparently everything is completely according to expectation: the students will eventually have at their disposal the same network as the teacher. The possession of a network of relations which is identical for all who make use of it and ideal for expressing reasoning -- a network in which all of the relations are connected in a logical and deductive manner; is this not the proper end of the teaching of mathematics?
>
> Let us not be too optimistic. First, a network of relations composed in this way is not founded upon the sensory experience of the students. Although it is possible that the network of relations itself has inspired some experiences for the student, the mathematical experiences that the student has been able to have are based completely on the network imposed by the teacher. This network, imposed and not understood, forms the basis of his reasoning. A network of relations which is not founded on previous experience risks, as we all know, being forgotten in a short time.

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* Isaak Wirsup, Breakthroughs in the Psychology of Learning and Teaching Geometry, in Larry Martin, editor, Space and Geometry, ERIC/SMEAC Center for Science, Mathematics and Environmental Education, Columbus, Ohio, 1976.

* P.M. van Hiele, La Penseé de l'Enfant et la Géometrie, 1959.

> Thus, the network of relations is an autonomous construct: it has no connections with the other experiences of the child. This means precisely that the student knows only what he has been taught and what is linked to it deductively. He has not learned to establish the connections between the network of relations and the real sensory world. He will not know how to apply what he has learned to new situations.
>
> Finally, the child has learned to apply a network of relations which one has offered him ready-made: he has learned to apply them in certain situations specially designed for him, but he has not learned to construct such a network himself in a domain as yet unexplored. On the other hand, if as a result of our teaching the students should obtain the capacity to construct a deductive relational network in a new domain, we will have achieved an optimal mathematical training. (p. 200) "

Unfortunately, we seldom feel that we can spare the time to build up theoretical constructs from a variety of practical experiences. Nor do we take sufficient account of the greater part of the individual's learning; that which takes place outside the formal education system and which is absorbed and assimilated from many situations, often repeated over long periods of time. Such knowledge is deeply embedded, by contrast to the patina of information that formal teaching often imparts. It is no wonder, then, that when our teaching conflicts with what has become "intuitive" the latter is so often resistant to either erosion or accretion. If school learning and intuitive understanding are not firmly linked we have a conceptual structure, with a crack down the middle, that will need much rebuilding before it is sound and secure. Some individuals have the aptitude for combining the two parts of the structure; all too many do not find this easy. So, since new knowledge can only be acquired by accretion onto the existing structure, we may be laying bricks in midair, a practice that is possible only on paper. In the real world, they fall down.

The development of concepts of spatial relationships are not the sole prerogative of geometry. Integration of subjects is notoriously difficult at the stage of specialization, but in the critical early stages, in the elementary school, this difficulty can be avoided. In working with maps, ideas of size and scale, position and coordinates, direction, and areas are inherent. In art and crafts, students may deal with relative size and position, with shape and perspective, with symmetry, with the projection of light and the highlights and shadows that result, with patterns and models, and with the representation of solidity in two-dimensional surfaces as well as the use of solid figures.

Above all in the physical sciences one is dealing with phenomena that require a knowledge of spatial concepts for their understanding; as our data indicate. Perhaps what is needed is a reversal of the usual order; we need a geometry learned from physics, rather than a physics based on an understanding of geometry. This would be a return to geometry as geo-metry, tackling life size measurements rather than small scale abstractions. It would reverse the old precept of "simple to complex," for reality is complex but young people live and make their way in a complex world and they can learn to abstract from this complexity, moving from what is to the underlying principles. Finally, it would be a dynamic geometry, in which the swing of a pendulum would lead to the concept of angles and angular measurement, balanced or hinged mirrors to symmetry, magnifiers to similarity, floating and sinking to density and volume. Outside of school these physical events really do come first.

There has been an enormous amount of study of the learning of geometry as geometry, as an important branch of mathematics, but little attention seems to have been paid to the critical importance of geometry as a basis for scientific understanding. Our research, which has only touched the surface of this problem, suggests that it is worthy of much more extensive and intensive investigations.

ECOLOGICAL CONTEXTS OF CRITICAL BARRIERS

ABRAHAM S. FLEXER

CONTENT

- Psychological Sources of Learning Problems
  - Inability to employ advanced strategies of reasoning
  - Inappropriate expectations
  - Trusting too much and too little
- Differences in Personal Learning Styles
  - Need for repeated, direct experiences
  - Need for structure
  - Reactions to being overloaded
- Problems With Pedagogenic Causes
- Concluding Statement

ECOLOGICAL CONTEXTS OF CRITICAL BARRIERS

This essay describes and documents an important distinction between critical barriers on the one hand and their ecological contexts on the other. The distinction emerged most forcefully as the project's staff constructed the preliminary taxonomy of critical barriers, presented elsewhere in this report. It became clear during that process that many of the problems encountered by students of elementary science cannot properly be ascribed to critical barriers, at least not if the power of that construct is to be preserved as originally intended 9see David Hawkins' essay). Moreover, we came to appreciate the significance of the fact that critical barriers aren't expressed in isolation, but rather in situations that include the students, teachers, and physical and psychological settings. Together, these elements create a context, an ecological setting in which critical barriers should be studied.

We have in mind an analogy with the approach a field biologist might adopt in studying and classifying the organisms of a newly discovered island. To a field biologist, organisms are most appropriately studied _in situ_: their behaviors with one another and their interactions with the inanimate surroundings are _in principle_ as important to investigate as their anatomy and physiology. Seen in this way, field biology includes a legitimate natural history component. Ethologists, anthropologists, and sociologists have repeatedly demonstrated the utility of this contextual or ecological approach to their respective domains of study. In recent years, some educational researchers, too, have proposed that such an approach would enrich research set in the classroom. We concur and take that approach here.

As used here, the rubric ecological contexts includes: subject matter; pedagogic techniques; biographies of students, teachers, and the class as a whole; students' conceptions about themselves, their worth, talents, and shortcomings; their attitudes toward science, learning, schooling, and teachers. Teachers bring corresponding sets of conceptions and attitudes. We argue that all of these, together with the physical setting of the school and class, contribute to the ecological contexts in which critical barriers are manifested. All need to be acknowledged if such critical barriers are to be fruitfully studied, dealt with, and overcome. This essay describes some of the factors that contributed to the ecological context of the course on Light and Color, and offers a primitive taxonomy of these ecological factors.

There is no intention here to reinvent psychology, nor do we suggest that critical barriers are largely psychological, except in the most trivial sense. Rather, we want to document the kinds of ecological factors that formed the context in which we observed the critical barriers elsewhere reported. As in my accompanying eassy on the taxonomy of critical barriers, we attempt a preliminary taxonomy, and again, we precede our proposal with a caveat. Some of the proposed categories overlap in ways not yet clear, and they are neither mutually exclusive nor exhaustive. We have not found all the joints*, but the following categories suggest where some of the important joints may occur.

TABLE 1. ECOLOGICAL CONTEXTS OF CRITICAL BARRIERS.

- Psychological Sources of Learning Problems
  - Inability to employ advanced strategies of reasoning
  - Expectations of self
  - Trusting too much or too little
- Differences in Personal Learning Styles
  - Need for repetitions, reflection
  - Need for direct experiences
  - Need for structure
  - Reactions to being overloaded
- Problems with Pedagogenic Causes
  - Inappropriate expectations
  - Poor analogies and representations
  - Confusing common and technical terms
  - Overloading students
  - Ignoring psychological difference
  - Not trusting students

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* David Hawkins, 1966. Taxonomy and information. *Boston Studies in the Philosophy of Science*, 3:41-55.

PSYCHOLOGICAL SOURCES OF LEARNING PROBLEMS

We include here a number of problems which are not, of themselves, sources of critical barriers, but may nevertheless compound difficulties caused by one or another critical barrier. That, indeed, is the essential distinction between the elements dealt with in this essay and the critical barriers considered earlier. Each of the problems described below originates within the learner, although each may be compounded from without.

Inability to Employ Advanced Strategies of Reasoning

Elementary science is full of analogies. Analogies are almost unavoidable as ways to deal with otherwise inaccessible objects such as solar systems, ecosystems, organ systems, cells, and atoms. Because analogies are so common, learners who are skilled at dealing with them, at extracting only what is directly relevant, are likely to have fewer difficulties than learners who lack such skills. Consider, for example, the use of a wire screen and balls of several sizes as an analog for osmotic phenomena. A student skilled at dealing with analogies might find this one helpful, although imperfect in many ways, most strikingly in the omission of the solvent. This particular analogy might be counterproductive for a student less skilled at dealing with analogies, precisely because the model omits the solvent in order to focus on the solutes and because the behavior of the solvent is an important key to understanding osmosis. The course on Light and Color demonstrated that learners of all ages may be inhibited by unhelpful analogies. Essays earlier in this report mention the ineffectiveness of using waves and particles as models, analogies for light; of comparing the eye, the pinhole camera, and the photographic camera; and of trying to demonstrate how shadows form by stretching a wire from a light source to a wall while keeping it tangent to the object projecting the shadow. In most of these instances, problems arose because the participants-- mature adults and teachers-- were unable to dissect the analogies into their useful and irrelevant components.

The ability to deal with analogies is only one of many strategies of reasoning that would be helpful to learners of elementary science. Others include the ability to apply proportional reasoning, to apply combinatorial analysis, and to infer logical necessity. These abilities are among those that many psychologists and others regard as being linked to the learner's intellectual development. Some researchers, for example, would assert that children younger than a particular range of ages are not likely to have yet acquired the capacity to perform these sorts of _formal_ mental operations; others are less persuaded of the developmental link. Whatever the reason, a learner who lacks one or another of these abilities _is_ likely to encounter difficulties with many aspects of elementary science, regardless of whether the difficulty involves a critical barrier. It is in this sense that we regard the inability to employ advanced strategies of reasoning as an important ecological factor to be considered by those studying critical barriers. By keeping this background clearly in mind, the role of the critical barrier can more effectively be assessed.

Inappropriate Expectations

It is a truism that our expectations may shape our experiences, at least in the short term, and students' expectations unquestionably shape their classroom experiences in important ways. Students of elementary science arrive in the classroom with rich and multipally connected conceptions of the natural world. Ways in which expectations based on these prior conceptions may interfere with learning are documented and analyzed in David Hawkins' part of this report. Here, we focus on other inappropriate expectations students impose on their classrooms. We argue that these expectations contribute to the ecological contexts in which students and researchers encounter critical barriers.

Students bring with them expectations about what should be included in the curriculum. Students in the course on Light and Color for example, had strong convictions about what should have been included in that course. Nearly all believed that the staff would sooner or later (preferrably sooner) have to explain the the colors of light in terms of light waves and their wave lengths, even though the staff had given them no reason to believe this would happen. Students expressed strong dissatisfaction when the expected connections were not soon presented. When the students insisted, the staff explicitly declined to present those connections on the grounds that they did not know how to do so except through a didactic lecture on atomic structure which would be out of place in the course. That stance generated more displeasure and disappointment than any other single feature of the course (see transcript of class of 4 November, 1981 in Vol. V of Raw Data, pp. 94-100)--a strong example of the ways expectations can establish what we are calling an ecological context. In response, a nontechnical lecture on the interactions of light and matter was presented in the following session (see transcript of class of 11 November, 1981 in Vol. V of Raw Data, pp. 101-107). Although this lecture was a concession to the students' expectations, reviews were, predictably, mixed, and continued to reflect the students' expectations for the course and for themselves.

> In some ways I was glad you plunged ahead with explanations to these types of topics which we didn't fully understand, such as light waves, yet I have found... that the explanations didn't stick.
>
> Polly, final paper
> Vol. IV, p. 279

> At certain times during the course, I felt angry about not being given more information. However, now I am grateful that I wasn't deluged by facts.
>
> Marilyn, final paper
> Vol. IV. p. 247

But clearly, the inappropriate expectation that the course would eventually "explain" light and color in terms of waves formed a very important part of the ecological background against which students and researchers operated, not always knowingly. Another group with other expectations might well generate a different ecological context, even with the same staff presenting similar topics in similar surroundings.

Students also bring to the learning situation expectations about the nature of science and about how science is to be learned. Consider these statements from members of the Light and

Color class about their expectations of what science is and what the truth is about:

> There is no coherence between physiologists' and psychologists' theories of vision and light. (That sounds crazy to me.) Neither one must have the whole ' truth-- the whole truth couldn't be in two split parts by nature-- perhaps.
>
> Hedy, notes of 10/28
> Vol. IV, p. 64

And there are expectations about how science is to be done, how it is to be learned. These, too, contribute to the ecological context in which learning occurs, as is evident in the following excerpts from the record of the *Light and Color* class.

> It seems important, too, to try to experience the materials, to try to observe what's happening as though one is seeing something for the first time, in other words, to try to see what's happening as it is without any preconceived notions...
>
> Diane, notes of 10/28
> Vol. IV, p. 60

> We have so many prejudices, pre-learned reactions, & preconceived ideas that it is hard to be objective, to see things as we *really* see them.
>
> Betty notes of 11/14
> Vol. IV, p. 88

> I expect to have lots of loose ends. This is what this [class] is all about and also the state of my understanding of concepts. I'm prepared for that to be the way of things, not only with the purpose of this seminar, but with how I operate in the world at this point in time.
>
> Jean, final paper
> Vol. IV, p. 252

Self-Image

Students also bring to class expectations about themselves, their worth, talents, and shortcomings. These, too, inevitably contribute to the flavor, the context of the learning environment. Indeed, the importance of what psychologists refer to as self-image is so generally ackowledged as to require little internal documentation. A sampling of extracts from the course on Light and Color will make the point that even among mature adult learners, these kinds of expectations can affect the dynamics of a class.

An importance difference among the students in the course on Light and Color, and the only one we document in this context, had to do with their beliefs about their own capacities to discover, to learn about science as they experienced it. Many felt confident of being equal to the task:

> ...now I am ready to go on... step by step--
> but each step says you are getting closer.
> ...once I've gotten something I think, "Well,
> what's next..."
> Polly, notes of 11/4
> Vol. IV, p. 68

> Things are clearing up very slowly for me... Next
> time-- it will break...
> Ken, notes of 9/30
> Vol. IV, p. 26

Other students were less sanguine about being equal to the challenges of the course.

> I agreed that it could be a possibility [that white
> light contains all spectral colors], as I had read it
> in a high school physics book. Yet that's not a
> discovery I fell I would have made myself as I am so
> conventional or "practical" in my outlook.
> Cindy, notes of 9/23
> Vol. IV, p. 27

> Again, I guess I am different. Being thorough is not one of my virtues...
>
> Betty, notes of 10/28
> Vol. IV, p. 62

> What is it that enables some people to say "OK, that's where it is for now" & work with what is known, whereas I just keep wanting to go back further & have difficulty working from what is known or I know? Wonder how much this has to do with my thinking that it's ... too difficult to pursue & that I'm afraid to go into the subject in depth?
>
> Jean, notes of 11/4
> Vol. IV, p. 75

At the other extreme was one conscientious student who was invariably an active participant and hard worker. She was, however, unconvinced of her own competence as reflected in the following two statements:

> I don't know if I could understand it [sources of color] or not or what I can understand and what I can't since I tend to assume that I can't understand lots so that# I'm willing to accept if a teacher says I can't.
>
> Hedy, notes of 11/4
> Vol. IV, p. 74

> I'd like to know more about those theories [of color vision] in one way [but] I have a reservation-- the feeling that I probably wouldn't understand them anyway.
>
> Hedy, notes of 10/14
> Vol. IV, p. 50

Moreover, she had so long been convinced to the contrary that science classes were frequently extremely discomforting:

> Being in such a frustrating situation brings up all my feelings of panic, being abandoned-- a lot of fear, an insurmountable problem, no means to approach it and no hope of ever having a way to solve it and no one to help. As extreme as that sounds and feels, when I feel like that I feel quite paralyzed and that it is very difficult for me to ask for help even if it was nearby. I am going into some detail on this because this is what I run into when math or science gets "too hard."
>
> Hedy, notes of 9/30
> Vol. IV, p. 179

G

The reader may easily imagine the interactions among individuals with such divergent attitudes about their abilities as students, and the ways in which these and other attitudes shaped the flavor of the class.

Trusting Too Much and Too Little

Finally, students have important expectations of their teachers. As the mentor, the guide, a teacher faces the challenge of providing enough guidance to encourage the student to develop at his or her optimal pace, yet not so much as to inhibit that development. Teachers need also to encourage a healthy degree of skepticism, while at the same time preventing habitual rejection of received knowledge. The extent to which a teacher successfully achieves these delicate balances is patently important, affecting not only attitudes toward information but toward learning as well. Again, direct quotations from the Light and Color course provide the best documentation of this obvious idea.

nnnnnn

Many students expressed gratitude for the trusting atmosphere developed at the Mountain View Center. The following is typical:

> The thing is when, as a student, I step off into a whole realm of questioning then I don't know if such lines of questions are in the right direction or not. Our naive or commonsense questions sometimes are in the opposite direction from understanding scientific truth. So when many explanations are far-fetched, or I am trying to understand far-fetched things, I am depending on a teacher to help point me down the fruitful paths.
>
> Hedy, notes of 11/14
> Vol. IV, p. 96

Trust, however, has limits, and is fortunately not by itself able to overcome strongly held, conflicting views.

> I also felt really interested in why light behaves so differently from pigment and pigment is all I have to relate to and that's where my "common sense" is and I felt like Ron was saying, "Just don't think the way you think." I feel like I can't let that go, that's all I know about.
>
> Hedy, notes of 9/30
> Vol. IV, p. 179

Indeed, too much trust, of teachers, of the power of received knowledge, can become counterproductive, stifling.

> The more I think about it, the less sense the experiment with the string makes to me. I realize, too, how gullible/naive I am. When a professor/teacher says-- "Now, this experiment will show you that light travel in a straight line."-- I believe it and my initial reaction is not to question the experiment.
>
> Diane, notes of 10/21
> Vol. IV, p. 46

> I never questioned. It says in my 4th grade sci. book that light travels in a straight line. I accepted that as a child accepts the faith of someone they admire and respect. I am changing. My mind is opening up more, since we started the class...
>
> Sue, notes of 10/14
> Vol. IV, p. 47

DIFFERENCES IN PERSONAL LEARNING STYLES

The second category of ecological factors we consider includes sources of learning problems which, like those of the previous category, are basically psychological and which originate within the learner. These problems reflect the fact that a given person may find different styles of learning most satisfying for different kinds of learning. A person who prefers to begin to learn a new sort of music by playing it may prefer to begin to learn a new handicraft by watching a skilled practitioner, and may prefer to consolidate early learning of both skills by reading how-to manuals. Such individual preferences are likely to change over time as skills, goals, and interests wax and wane.

None of that is novel. Anyone who has learned across a variety of domains, or who has been taught in one domain to a class of any size, is aware of these differences in personal styles and the ways these affect individual learning. Skilled teachers are also aware of the effect on the social dynamic of a class of the sum of these individual differences. We document below some of the differences in this regard observed among the participants in the course on Light and Color. We do so because mature adults display these preferences as often as do children, but are often better at describing and expressing them and at evaluating their significance. Receiving comments from adults, such as those quoted below, clarified for at least some of the staff many previous observations and experiences with young learners. These extracts, and others, suggest to us that there are at least three areas in which the learning styles of the participants differ in noteworthy ways (Table 1, above).

Need for Repeated, Direct Experiences

Most of the participants expressed a preference for direct experiences as opposed to didactic instruction, for opportunities to handle objects, to manipulate apparatus.

> Setting up the equipment [pinhole camera] made it sink in, and I don't think I will forget again...
> Betty, notes of 11/18
> Vol. IV, p. 102

> The important thing to me is to do the experiment myself, have some time to think it through, rather than sitting and watching someone else do a demonstration.
> Diane, notes of 10/28
> Vol. IV, p. 61

> I need more time to use gels and lights and time to do it slowly and do it step by step and write down observations.
> Diane, notes of 9/23
> Vol. IV, p. 15

Merely reading or hearing about a phenomenon was not usually regarded as an adequate learning style, as expressed in this comment:

> Today certainly reinforced my knowing that I must have a conscious experience with a phenomenon to "know." Even though I have read David's article, I still did the mirror reflection drawing as though I'd never had any exposure to this whole idea. Obviously, I'd not sat down to make the mirror idea make sense to me while I was reading.
> Jean, notes of 11/14
> Vol. IV, p. 87

In addition, many students expressed a need to repeat experiences, to try manipulating the apparatus more than once, prehaps in different ways.

> ...being exposed to a concept in a variety of ways is extremely helpful... it's helpful to repeat the same experience in the same way as well as to having a similar experience in perhaps a somewhat different way.
>
> Diane, notes of 11/1
> Vol. IV, p. 62

> We probably did this [observed white light where light from colored gels overlapped] 3 or 4 times. I really needed to see it redone. And when I form the image of that sequence in my mind, I still am marvelling--that I can see that we made "white light" on the wall while in another way, I still can't believe it. Like an abyss on both sides of my little glimmer of understanding.
>
> Hedy, notes of 9/23
> Vol. IV, p. 23

Most students also wanted time to review what they had done, to reflect on what it might mean, to piece things together.

> Again, I feel I need more time to use the materials and draw conclusions on my own. I think after a certain amount of time and overhearing and joining in on various conversations, I'm saturated for the moment and need to come back to thinking about absorbtion and reflection of light another time.
>
> Diane, notes of 11/4
> Vol. IBV, p. 68

> I just needed to hear things more than once, or to hear them in another context, or to have some exploration time and to hear them again, or to have some discussion time and then hear again with different ears.
>
> Jean, final paper
> Vol. IV, p. 252

These comments are the more remarkable when one recalls that they refer to a course in which open exploration was encouraged and the pace was far more deliberate than in most undergraduate, or even secondary school, courses.

These preferences, which reinforce much conventional pedagogical wisdom about the benefits of repeated, first-hand experiences with concrete materials, were not, however, universally shared as the following suggests.

> ...I do not like going over the same things again and again. I realize, and [staff member] also told me, that the rest of the class does not feel this way. Again, I guess I am different. Being thorough is not one of my virtues. I get interested when things change, when I can go fast in my learning...
>
> Betty, notes of 10/28
> Vol. IV, p. 62

Need for Structure

Participants in the Light and Color class also displayed a remarkable range of preferences with respect to how much structure and guidance they wanted, how much ambiguity and openness seemed comfortable. As established in the preceding section, the style preferred by most participants was to explore slowly, to repeat experiences and observations as often as seemed productive. For many, this preference went along with a high tolerance of ambiguity, of uncertainty. These preferences for open exploration again reinforced many expectations shared by the project's staff. But again, such preferences were not shared by all the participants. Some disliked ambiguity and uncertainty.

> Color & light are fairly elusive concepts for me. I enjoy the interchange of ideas about them, but I seek one comprehensive explanation. Not knowing why or how light & color are explained is troubling to me. On a certain level I experience a "need to know" about them.
>
> Ken, notes of 9/23
> Vol. IV, p. 23

Others, either not skilled at open exploration or uncomfortable with that style for other reasons, disliked the prevailing approach.

> I have trouble within myself exploring a concept more than just a surface explanation. I appreciate those who "lead me on" and help me get further into the search. I feel that without these people, I

> piece of knowledge, and then not know how to proceed with further exploration.
>
> Anne, notes of 9/23
> %23

> I need some framework. I'm not content with a total inquiry approach. I like to try to find answers, but I also like being led to decisions and conclusions. ...in addition to the opportunity to discuss together as a group, I would like to hear a summing up of what we were supposed to discover.
>
> Sue, notes of 9/23
> %19

From time to time, several students expressed mild anxiety when observing other groups doing activities different from the ones they were engaged in. Their concern centered on "missing something" that the others were privy to, of wanting everyone to share the same experiences and information. And at least one participant stated a preference for a quite different style, at least for part of the time.

> Contrary to some other comments that you have received, I really like and need some time devoted to instruction, input, discussion, whatever. I find that more beneficial for me, than to have a total time that is all exploring and doing.
>
> Sue, notes of 11/11
> Vol. IV, p. 81

Reactions to Being Overloaded

At one time or another, almost every student experiences the sensation of being overloaded, overwhelmed by the pace of instruction or some other aspect of schooling. Too many instructions are imposed, too many variables are introduced , too many new ideas are broached-- all in too short a time for the student to accept. We are not concerned here with the particular circumstances that generate those feelings, or with the different thresholds at which students succumb. Although clearly important, those issues are tangential to the issues we address here.

Our observations of the participants in the course on Light and Color suggest to us that mature adults experience this sensation and that their strategies for coping with it are varied. We find in adults' strategies useful insights into how younger learners may react to similar feelings in similar situations. We explore this matter because we believe that the reasons learners come to feel overloaded, and the ways they react to that feeling, contribute importantly to the ecology of learning situations, the settings for critical barriers. We begin by presenting some typical examples from the Light and Color course that express the kinds of concerns we deal with below.

> Confused-- Think I don't want to try to figure out anything right now [end of class].
> Jean, notes of 9/30
> Vol. IV, p. 25
>
> I remember that after a time I felt I wasn't absorbing any more [of the lecture on light and matter]. Then I stopped writing. Then later I could focus again.
> Jean, notes of 10/21
> Vol. IV, p. 59
>
> The questions that were put on the board [because participants wanted "answers" during the course] were so difficult and some of them I never would have even thought about, much less ask. ... Sometimes the mind just doesn't seem strong enough to handle all this variety.
> Myhra, notes of 10/28
> Vol. IV, p. 63
>
> ...the unanswerables approached... That's part of why I feel frustrated to have to rush over it... I didn't have any interest in the new questions. Too content with my new glimpses.
> Hedy, notes of 10/28
> Vol. IV, p. 66

The point is that even adult learners reach a point of saturation, a point at which they close themselves to additional information or experiences. These excerpts also hint at the variety of reactions to feeling overloaded and at the range of ways in which the participants dealt with that feeling. The following excerpts document the variety.

At least some students who recognized being overloaded simply reported their awareness of that fact.

> I really have no answers and I'm OK with that.
> Cindy, notes of 10/7
> Vol. IV, p. 35

> And again, the wavelength idea is still one I would like to play with-- but I'm not in a hurry.
> Polly, notes of 10/14
> Vol. IV, p. 45

> Not interested in new questions yet. I will be.
> Hedy, notes of 10/28
> Vol. IV, p. 61

> ...some of my feelings of just letting the subject and my confusion from last time lie may have to do with some of the frustrating I felt [with the equipment]... so now I'll just need to hang in there until something develops that gives me a different enough approach/perspective that I can create another symbol/idea way of thinking about [light]...
> Jean, notes of 9/30
> Vol. IV, p. 32

> Not truly understanding light waves, is another thing to which I have resigned myself. I feel overall that I have learned so much therefore it it is acceptable to leave a few things hanging.
> Polly, final paper
> Vol. IV, p. 279

Other students who were aware of being overloaded also seemed aware that this was no cause for undue concern. They expressed varying degrees of confidence,even optimism, that the condition would pass or be dispelled.

> Some of the lecture zoomed right over my head and yet I still have an idea of what may have been explained which is one step closer.
>
> Cindy, notes of 11/18
> Vol. IV, p. 102

> Just because I couldn't follow a particular part of an explanation does not mean I don't want to hear it-- I don't want to miss the gems, and later I may understand more.
>
> Jean, final paper
> Vol. IV, p. 252

> Sometimes I wonder why I'm trying. I'm missing so many pieces-- even some pretty basic math-- that it's such a long road to understanding what is going on with such familiar subjects. I'll stay with it cause I will and basically I'm more optimistic than that-- just so discouraging sometimes.
>
> Jean, notes of 9/30
> Vol. IV, p. 33

Still other students became anxious, apprehensive, deeply troubled at being overloaded.

> New ideas impinging feels oppressive. Feeling some "so what." Not too excited-- barrier to?... Feeling dumb... I think I am facing my own lack of understanding-- and when I encounter that-- hand-in-hand it is I fear I can't understand. Diffuse.
>
> Hedy, notes of 11/4
> Vol. IV, p. 69

> There are a lot of pieces of information floating around for me just now. I'm wanting to hook them all together but I'm finding this to be difficult.
>
> Ken, notes of 11/4
> Vol. IV, p. 75

> It's almost as if your mind goes out of gear. You think you are internalizing everything and then all of a sudden you go back to a primitive way of dealing with information. I felt very confused and lost... For the first time I had real empathy with those children who become so upset when you're talking about regrouping using 3 and 4 digit numbers in subtraction, or the children who cry and say I knew my X tables last year.
>
> Sue, notes of 11/14
> Vol. IV, p. 90

> At one time,... I needed not to be around the class or to come up to Mt. View. ...I was particularly feeling confused with putting together what I was

> observing and hearing... It was such a dilemma for me at the time. I felt I needed time to absorb and not to work that week with equipment, etc., yet I knew I would be missing so much and would feel like I wished I had gone anyway. Sometimes I go and it works fine for me to be there.
>
> Jean, final paper
> Vol. IV, p. 253

Of necessity, we omit the crucial issues of how these thresholds and reactions are established and of how, for a given student, they change from time to time and from topic to topic. Nor can we consider ways in which teachers may become more aware of these phenomena or how teachers might better deal with them. Although those issues are undeniable central to improving schooling, limited resources preclude dealing with them now. We close this section by summarizing our theme: the kinds of feelings and reactions documented here also form an important component of the ecology of classrooms.

PROBLEMS WITH PEDAGOGENIC CAUSES

The final category of learning difficulties that we consider results from factors external to the learner. In this case, we concentrate on learning problems that have their loci in the the teacher or the teaching situation, or in the student-teacher interaction. We limit ourselves to a set of problems we call *pedagogenic* (caused by a teacher) in parallel with the established medical term, *iatrogenic* (caused by a physician). Both terms refer to well known experiences: teachers and physicians attempting to alleviate an undesirable condition may, inadvertently and unintentionally, generate other, equally undesirable, conditions.

The domain of inquiry here is enormous, at least as large the entire domain of critical barriers. Because this project emphasized its mission to investigate critical barriers, and because the staff concentrated on other matters, we were less aware than we might have been in other circumstances of pedagogenic problems we were causing. Nevertheless, the record brought some of these to our attention, sometimes forcefully. We mention below a few kinds of pedagogenic problems encountered during the *light and Color* course.

We have already considered the importance of analogies in elementary science classes, and pointed out the kinds of difficulties that students encounter before they acquire the skills to deal with analogies. Here we want only to point out that the inadvertent use of poor analogies can only exacerbate those difficulties. A single example suffices:

> You drew around a shadow and you said "there, that proves light travels in a straight line." I thought not to me, it doesn't. The paint proved it for me, because then I saw what I was closed to...
>
> Sue, notes of 10/28
> Vol. IV, p. 47

This points out the value of the truism that teachers should use only analogies that prove to be effective, Moreover, it illustrates the value of using more than a single analogy to make a given point.

In a similar vein, pedagogenic problems may arise when teachers inadvertently use common terms to convey special, technical meanings. Often, the technical meaning is not made sufficiently explicit, with predictable consequences.

> Does it [light] travel? I don't think of it as something that moves. I think of it as just being there where ever it is and there is more or less of it.
>
> Hedy, notes of 9/23
> Vol. IV, p. 7

> "White light" seems to be a misnomer-- really means clear light, natural light.
>
> Hedy, notes of 10/7
> Vol. IV, p. 40

Every elementary science course is replete with such misleadingly used common terms: _fruit_ and _vegetable_; _acceleration_ _sensu_ physical science; _cell_, as used in biology, physics, and chemistry; and so on.

Other, more general pedagogenic problems may occur when teachers have inappropriate expectations of what students are able to do. The first quotation suggests what may happen when students, adults in this case, are offered too little guidance. The second illustrates an outcome of expecting more than students can deliver.

> Not sure where to start. Appreciated being pointed to something by [staff member]. Not self-evident where the starting points were. Must often look like that to my kids-- tables with stuff on them-- "What do I do? Where's the start?" Wasn't really aware of that til I'm starting to write just now... Didn't see initially where that [eyedroppers & colors] might lead. Again must be how the little ones feel when equipment is set out and no questions are raised and there it is. It may be intriguing or not.
>
> Hedy, notes of 11/11
> Vol. IV, p. 83

> "You're supposed to know this by now." That's probably the greatest deterrent to acknowledging where you are, comfortably. That's restated many times in a growing person's life. Part of the overall message: "You're supposed to fit the curriculum," not vice-versa. And it truly doesn't serve learning.
>
> Hedy, notes of 12/2
> Vol. IV, p. 112

In addition, pedagogenic problems may arise when teachers fail to allow for the kind of personal differences described and documented in the earlier sections of this essay.

Most generally, pedagogenic problems may arise when teachers fail to trust their students, to attend to the students' concerns as legitimate causes for concern, and as important clues to other problems lurking beneath the surface. The following statement by a member of the *light and Color* class expresses this matter eloquently.

> I am continually aware-- increasingly aware-- of the importance of careful listening to the language youngsters are using and to what they are meaning by what they are saying. I try to assume less and to understand their points of view-- especially when it seems very different from mine.
>
> Jean, final paper
> Vol. IV, p. 255

CONCLUDING STATEMENT

The ebb and flow of activities dictated by the conflicting preferences and attitudes of the three general types documented in this essay helps to establish the dynamics of any class. Note that, at least in some cases, the preferences are incompatible, even mutually exclusive. Given that, the skilled teacher might attempt to accomodate the differences rather than seeking a compromise that may benefit no student for more the part of the time. This ebb and flow is what determines the ecology of the class and provides the backdrop against which critical barriers develop. The project's staff tried to be aware of them as we interacted with the participants in the course and as we analyzed our observations and other data. Those who wish to replicate or to extend this investigation of critical barriers would do well to recognize and allow for these and other factors separate from critical barriers, but not independent of them, for these establish the ecological setting in which critical barriers occur.

CLUES FROM THE HISTORY OF SCIENCE

ABRAHAM S. FLEXER

CONTENT

Biographical Sources of Critical Barriers

Historical Sources of Critical Barriers

"Non-Commonsense" Evolution of Scientific Theories
    Genetics
    Cosmology
    Structure of matter
    Classification of organisms

Biography and History may Generate Similar Theories

Interpretation and Implications

CLUES FROM THE HISTORY OF SCIENCE

Other sections of this report establish the range of circumstances and conditions in which critical barriers become evident. Here, we argue (1) that learners and teachers encounter critical barriers for biographically and historically valid reasons; (2) that the history of science is a source of insights into some of these reasons; and (3) that these insights can inform the teaching of at least some topics in elementary science.

BIOGRAPHICAL SOURCES OF CRITICAL BARRIERS

Students arrive in the classroom with rich understandings of the natural world, understandings that are self-consistent and useful. Students construct from their experiences and reflections a set of theories, or at least conjectures, which serve to make sense of their past and present and also to predict aspects of the future. Life would seem chaotic without such predictive theories, and every parent and teacher has observed children form and use such theories.

A little thought brings to mind many examples of such personal theories: that motion implies a persistent force; that the phases of the moon are caused by the earth's shadow; that weight is caused by the press of our atmosphere; that only the wick of a candle burns; that plants get most of their nourishment from the soil; that there are two thermal fluids, hot and cold; that with enough practice, one could step on one's own shadow; and so on.

For the most part, these personal theories are acquired through combinations of experience, reflection, and synthesis. Some, however, are acquired as received knowledge: "Mommy says,..." or "Teacher says, ..." In either case, the objects and phenomena to which these theories apply are generally those of everyday experience: toys and belongings, family and friends, the neighborhood. The ways in which learners (of all ages) form these experientially-based theories, or accept revealed theories, is the domain of developmental psychology and other cognitive disciplines, and we need not elaborate or speculate on what they might be. Only two points need to be made here about these personal theories. First, they are likely to be strongly held, often because they are repeatedly reinforced by routine experiences, usually the very experiences they rationalize. Second, they do not always coincide with adult or conventional views that youngsters encounter during formal schooling.

Personal theories do, nevertheless, change over time, in part because the experiential base to which they bring order expands, and in part because of educational challenges. Such changes are likely to be slow, however, because for most persons most of the time experiences sufficiently dissonant (or at least novel) to require modification of a useful personal theory are rare. And personal theories are likely to persist even in the face of dissonance and education.* It is this enduring quality of personal theories that may generate critical barriers when, in formal learning situations, students are expected in a short time to modify or even to relinquish their personal theories in favor of conventional theories which may seem to them less useful, indeed, less valid. Note that it is often the case that the conventional theory seems less fruitful, less powerful and interesting, than the personal one. For example, the conventional theory may apply to phenomena that are perceived to occur only in the formal setting, and are thus less significant, less in need of rationalization, than those already accounted for by the personal theory. Accounts of weight based on notions of universal gravitation described by an inverse square law, or of a moon constantly falling toward earth but never getting closer, or of plants building themselves out of thin air, understandably may seem arcane, anti-commonsensical.

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* See J.D. Herron, 1975. Piaget for chemists: explaining what "good students cannot understand. J. Chem. Ed., 52:146-150; M.M. Milkent, 1977. It's time we started paying attention to what students don't know. Sci. Ed., 61:409-413; J. Nussbaum, 1979. Children's conceptions of the earth as a cosmic body: a cross age study. Sci. Ed., 63:83-93.

HISTORICAL SOURCES OF CRITICAL BARRIERS

One can draw fruitful analogies between the kinds of biographical events that lead to the development of personally useful theories on the one hand, and the evolution of the conventional theories of the natural sciences on the other. For example, two of the hallmarks of theories in the natural science are that they are constructed (at least in part) from past experience and that they are continually validated by testing them against present, and predicted future, experiences. A particularly cogent analysis of this process is presented in David Bohm's analysis of the similarities between the growth of scientific concepts and cognitive development as construed by Jean Piaget.* Some of these analogies illuminate the observations made in the course of this project's research and suggest potentially useful teaching strategies as well as avenues for future research.

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* See D. Bohm, 1965. The Special Theory of Relativity, W. A. Benjamin, New York, 1965, pp. 185-230 .

Contemporary scientific concepts that form the content of elementary science have evolved and endured over decades, even centuries. The history of science reveals that many current theories first emerged from experiences and observations of objects, phenomena, and events that were, long ago, routine parts of daily life. The earliest scientific theories dealt with objects and phenomena that were generally accessible to most persons most of the time: motions of heavenly objects, tides, similarities among parents and progeny, motions of projectiles, and so on. Those theories can be easily traced to common experiences that could be shared with others; many are just as accessible to children and adults today.

Historians of science do not agree about the stimuli that generated successive modifications of very early scientific theories. Indeed, it seems likely that different kinds of stimuli operated in different domains at different times. Yet the sources of at least some historically important, generic stimuli are clear. Progressively closer observations of familiar objects, progressively more penetrating experiences with common phenomena, revealed new features and regularities not at first accessible to the casual observer. Archimedes, Galileo, Kepler, Harvey Pasteur, and a host of other historically germinal figures offer case studies of such intense scrutiny of common objects. And often, a prevailing theory had to be discarded, or at least significantly modified, in order to accommodate the new insights. Repeated cycles of deeper experiences that required successive modifications yielded progressively more powerful scientific theories. These cycles also led to unexpected (though understandable) consequences that are relevant in the context of this project.

Seen in this way, modern scientific theories evolved in directions that were determined when some observers looked at common objects and phenomena in new, more intense ways; when other investigators attempted to solve problems that were not part of the daily routines of ordinary people. As the novelty and intensity of observing deepened, the observers began to have experiences not generally accessible to others. Science began to answer questions that ordinary people could not even ask, and to offer answers increasingly incomprehensible in everyday terms. Explanatory schemes shifted from the macroscopic realm to the microscopic or invisible (see next section). A symptom of this experiential gap is today observable in the laboratory manuals used in many elementary science courses. One of the functions of the laboratory portions of such courses is to provide the neophyte opportunities to recognize the questions to which theories presented in the lecture portion of the course provide answers. Laboratory "experiments" are needed precisely to provide students with otherwise inaccessible experiences.

Many examples can be cited of this progression from theories that account for common phenomena in terms of readily experienced entities to theories that account for obscure phenomena in terms of entities too large or too small for the naive learner to conceive of. The following examples are illustrative.

"NON-COMMONSENSE" EVOLUTION OF SCIENTIFIC THEORIES

We sketch here some examples of the ways in which particular topics in modern science have evolved in directions progressively divorced from common experiences, progressively alienated from common sense. Four examples will establish the pattern, although many others could be cited.

Genetics. The understanding of heredity evolved from a set of correlations that focused on macroscopically visible traits of parents and progeny to a reductionist view of organisms that focusses on invisible informational macromolecules. Whereas in earlier days the subject matter was accessible to the average person, contemporary genetics is accessible only through molecular biology.

Cosmology. The transition from the belief in a flat earth of limited size, through geocentrism to heliocentrism occupies an important place in Western intellectual history. Most of the data relevant to that sequence was accessible to naked-eye observers. But from the time of Tyco and Kepler forward, astronomical and cosmological sciences have evolved to accommodate observations and data obtained through increasingly sophisticated and inaccessible instruments: first through several generations of progressively more powerful optical telescopes and spectroscopes; later through progressively more sophisticated radio telescopes and other electronic devices. The recent images returned by Mariner and Voyager vehicles are formed, transmitted, and reproduced by means incomprehensible to more than a small fraction of the population.

Structure of Matter. Notions of the indivisibility of matter's ultimate constituents date at least to the Hellenic Greeks, but were closely connected to the observable properties of the basic elements, easily experienceable and observable. Historically intermediate views, also based largely on observable properties, differed mainly in the number and interconvertability of the fundamental elements. But modern views of the atomic structure of matter violate common sense in requiring that much of the volume of all materials be void and that even atoms are resolvable into entities that may, in the end, not be material in the commonsense meaning of that term. This set of ideas, again, derives from data obtainable only by the few through large and expensive machines.

Classification of Organisms. In ancient times, an educated person could assign each organism unambiguously to one of a small number of unchanging categories defined by a few straightforward criteria. Linneus introduced additional, often subtle, criteria and defined new kinds of categories, but retained the earlier notion of immutable species. Post-Darwinian schemes introduced evolving species and became explicitly phylogenetic; still, most criteria were still macroscopic, readily accessible to educated laypersons. Modern phylogenetic taxonomies have added to post-Darwinian criteria other criteria, including ultrastructural, biochemical, ecological, and behavioral features that are, once again, accessible only to specially trained individuals.

Note that all these radical shifts date from recent times. Major changes in genetics began in the mid-20th century. The evolution of species, and the existence of atoms, were questioned by reputable scientists as recently as the turn of the century. In each case, and in western intellectual history generally, the divergence of common sense from science has been clear, overpowering. The divergence reflects and is driven by the very different commitments and goals of common sense and science. Common sense seeks practical ends, ends often achieved by knowledge and solutions that satisfice under reasonable circumstances, even if they are not perfectly generalizable. Science, by contrast, requires strong, cohesive, and minimally redundant knowledge that is also maximally generalizable. Given these radically different commitments, the divergence is inevitable and should not be surprising.

BIOGRAPHY AND HISTORY MAY GENERATE SIMILAR THEORIES

It seems clear, then, that modern science deals with many objects and constructs no longer accessible to ordinary experience, and accounts for phenomena no longer accessible in the daily lives of most persons. It answers questions that few nonscientists recognize as meaningful, and answers them in ways that even fewer can understand. Yet, outside of formal educational settings, most people, particularly children, continue to construct theories about the world more or less directly from their personal experiences with readily accessible objects and phenomena. This is inevitable and not to be discouraged or even regretted.

We raise these issues because we think that students of elementary science may retrace in miniature some early steps in the evolution of current theories. Given that nature seems little changed since the times when earliest scientific theories were formulated, it seems reasonable to expect that today's naive observers will derive from their experiences personal theories that will resemble early scientific theories. These personal theories are likely to conflict with conventional theories when those are introduced through formal education, and perhaps conflict in ways that will lead to critical barriers. This may happen because learners faced with a conflict between personal and conventional theories need not only comprehend the novel theory, but must also integrate-- reconstruct-- their prior knowledge and experience to be consistent with the new view. This kind of reconstruction, whether revolutionary or evolutionary for any individual or theory, takes time and hard work. Until the reconstruction is demonstrably preferable, the learner is likely to resist it, retain the personal view, and encounter critical barriers.

One can test this line of reasoning by examining the range of personal theories revealed in situations likely to elicit strong cognitive dissonance between personal and conventional theories. To the extent that the personal theories resemble early scientific theories, the argument would be supported. The courses offered by the project's staff provided just such situations, as is documented in the other essays in this report. Ideas about hot and cold thermal fluids were challenged by observing dry ice boil liquid nitrogen. Ideas about the nature of light and color gained from mixing pigments were challenged by experiences with light filtered through colored gels and by observing images formed in _cameras obscura_.

The course on _Light and Color_ was a particularly rich source of personal theories with historical resonances. Reading the record of that course (Volumes IV and V of the Raw Data) reveals a transition that occupied science for two millenia: from the emission theories and *lux* of the ancient Greeks to the *lumen* that flows from Kepler's decoupling of the external and subjective and paved the way for geometric optics and Newton.* This decoupling of the observer and the observed was reenacted quite remarkably in our course.

INTERPRETATION AND IMPLICATIONS

Based on our observations, students' personal theories *do* resemble early theories from the history of science. And, at least in some cases, students were helped to reconstruct their experiences and to adopt conventional theories by being confronted with credible, dissonant experiences. Although our data are not adequate to settle the issue, we feel that these reconstructions were facilitated when the dissonant experiences were analogous to those that induced the historical reconstructions which punctuate the history of science. Put briefly, we are asserting a recapitulationist argument: the development of personal theories recapitulates to some extent the development of early theories in the history of science.

See the early chapters of V. Ronchi's *The Nature of Light: An Historical Survey* (translated by V. Barocas), Harvard University Press, Cambridge, 1970.

We argue, therefore, that science educators might do well to look to the history of science for precedents. It may be possible to anticipate the kinds of personal views their students are likely to bring to class, and to anticipate the kinds of dissonant experiences to offer students when they are are on the verge of making transitions to conventional theories. Seen this way, good pedagogy would include being receptive to students' personal theories, their native frameworks, in order to anticipate how to help them recognize the need to reconstruct their ideas. Good pedagogy needn't involve teaching away students' theories. Rather, teachers could use those theories to make students aware of counterexamples that encourage them to reconstruct their knowledge. Rather than devaluing or ignoring personal theories, and thus devaluing the student, skilled teachers might encourage the student to have experiences that will predispose them to make the desired reconstruction. This has the additional advantage of allowing the student to recognize that the world is knowable, understandable, and that science is an effective path toward that understanding. That is, after all, one of the underlying goals of science education.

It needs to be said that we are not advocating teaching the history of science in elementary school. There is no point in teaching the historical development of science merely to unteach it in presenting the contemporary view. Nor are we advocating that teachers of elementary science become historians of science. The training of future teachers is already too rushed.

We are advocating that teachers become aware of the historicity of modern scientific theories so they can recognize "old" ideas that their students devise as they attempt to rationalize experiences of the kind that led originally to the old ideas. We believe that courses similar to those taught under this grant would be appropriate vehicles by which teachers could become aware of the historicity of scientific theories and of their significance and potential usefulness in their own classrooms.

MAN AND NATURE - ENERGY COURSES

Fall Semester 1980

Spring Semester 1981

EVALUATION

by Ronald Colton

MAN AND NATURE - ENERGY COURSES

Fall Semester 1980

Spring Semester 1981

During these two semesters we participated in teaching and observing two science courses for non-science majors offered in the Center for Interdisciplinary Studies with which Mountain View Center is affiliated.

The Courses

The courses were entitled, "Man and Nature - Energy", described for students in the following terms:

Catalog Description

The general theme of "energy" is used to focus attention on man's relationship with his environment. This is an interdisciplinary, integrating, liberal arts course to show the relationships among the natural sciences and to suggest how science helps us to understand our surroundings. The course provides an opportunity for student research, writing, and field and laboratory work in the context of a socially relevant subject-matter theme. The course is designed as an elective for non-science students and an integrative science course for prospective teachers. It fulfills the natural science area requirement for A & S students ("first-level" course combination).

General

This is a course about science in general and about energy in particular. It is an experiment. Nothing quite like it ever has been taught before at the University of Colorado (or anywhere else, as far as we know). We trust that you will enter into the spirit of the experiment by providing continual commentary on what you find useful and what you do not. Let us know what you would like to investigate that seems to be neglected on the schedule. Share you concerns with us. Let us know when you are confused.

And don't be afraid to say so. You may be the only one who will admit it, but surely you are not the only one who is confused!

The course is an experiment in another sense. Each of the instructors is interested not just in science, but in the teaching of science. We want to know how to teach better and what sorts of concepts and techniques pose difficulties for learners. Hence, a deliberate attempt will be made to approach concepts from a variety of perspectives and we instructors will observe each other, and students, as we attempt to communicate.

We are committed to the principle that science is a process important in people's lives. Science is not so much a list of facts as it is a process for arriving at an understanding of the way the world works.

The course is planned to have a variety of functions. Among them are:

* to inform students about energy as a phenomenon in the natural world and in the daily lives of individuals;
* to communicate something of the nature of the process of science;
* to break down the over-drawn distinctions between the sciences (geology, biology, physics, for example) and between the sciences and other fields of human endeavor;
* to fulfill the Natural Science requirement of the College of Arts and Sciences in a single semester.

Each course was team taught by three instructors. In each there were about 75 students, sharing three 50 minute lectures each week and divided into three laboratory discussion groups of about 25, each in charge of one instructor. Students ranged from freshman to graduating seniors though the majority were sophomores.

In the first semester one project member acted as instructor and another participant observer of the lectures and one laboratory/discussion section. During the second semester two project members acted as instructors, sharing one laboratory/discussion section.

The course involved a considerable amount of quantitative work, though this was almost entirely elementary arithmetic. Students were expected to be able to work simple problems involving conversions between Kcal, KWh, Btu's, heat loss and insulation, personal energy use, cost comparisons between various fuels, effectiveness of solar flat plate collectors, etc.

Problems Encountered

The overriding problem was that of the difficulty that the majority of students had in solving problems involving elementary arithmetic. This was coupled with a reluctance to estimate approximate answers, or even to appreciate the value of this and students' extreme discomfort in accepting "order of magnitude" calculations when these were presented during lectures and discussions.

Specific Difficulties Encountered

1. Confusion between fission and fusion reactions was common.
2. Terms for invisible particles often had little meaning and students failed to distinguish between electron, atom, molecule, etc., for example, "atoms surrounding the nucleus".
3. Similar confusion between cells, molecules, etc.
4. Lack of productivity in deep oceans often attributed to lack of light and not lack of nutrients.
5. Confusion between "greenhouse effect" and destruction of ozone.
6. Lack of a "fractorial sense," so that students failed to see relations between say, 8, 32, 64, etc. and so carried out needlessly lengthy calculations.
7. "In exponential growth the doubling time increases at an increasing rate."

8. "Clouds get heavy and make it rain."
9. While the water cycle was fairly easy to understand, some students failed to understand atmospheric transport and felt that water in the University pond evaporated, rose into the atmosphere, condensed and came down in the same place.
10. The carbon cycle posed difficulties - students found it difficult to understand how a black solid like carbon could become air, invisible gas, which in turn could be converted into plant materials.
11. Latent heat. "The energy is absorbed over the oceans. This energy is released over land in the form of rain."
12. "The ultraviolet rays entering the atmosphere bounce off the earth as infrared rays."
13. Confusion over the 1st and 2nd laws: "energy goes from a high form to a low form or that energy is lost."
14. Tides caused by the sun, for example, the greater effect of the moon was not understood.
15. Difficulty in experimenting with small demonstration solar collectors. Some students found it hard to understand why when the volume of water was measured, it was also necessary to measure the area of surface exposed to sunlight.
16. "Energy loses 10% of its efficiency at each stage down."
17. Photosynthesis and respiration - roles of CO_2 and O_2 caused confusion.
18. "For example, when the sun's energy is converted to kWh an incredible amount of energy is lost in the process."
19. Hot air is confused with heat, for example, it is identified with heat.

20. Objects weigh less on a mountain because the air pressure is less.

21. Difficulty in moving between heliocentric and geocentric views of the earth's motion.

22. Difficulty in visualizing celestial motions in 3D planetarium.

23. Difficulty with the notion that if A = B and B = C, then A = C.

24. kW and kWh, that is, rate and quantity.

25. Reluctance to accept that when a tank of compressed O_2 was connected to a calorimeter, the pressure in the latter would rise to that in the tank if not controlled.

26. Difficulty with the concept of a vacuum in a thermos flask.

27. Mg weigh more after burning $Mg + O_2 = Mg\ O_2$.
"But O_2 has no weight - I mean you can't weigh it."

28. If you put a bucket of boiling water in the pond it would not alter the temperature even in theory.

29. Water escapes from an inverted jar when air is admitted because the air pushes it out.

30. Temperature gradients were found difficult to visualize.

31. Non-linear growth and decay.

32. Relative humidity, relative density.

33. Projection of equator to give celestial equator.

34. Area. 10 X 10 = 20. 10 square centimeters and 10 cm. square.

35. Series and parallel in electrical circuits.

36. Heat and temperature.

37. First Law states that energy is recycled so none is lost.

38. Light years.

39. Rainfall in inches.

40. Relative motion.

41. Effect of altitude on boiling point.

42. The fact that 1cc of water weighs one gram causes confusion between mass and volume.

43. Concept of work. One may be working hard standing still holding a load.

44. Specific heat.

45. Conversion of °C to °F and vice versa. Students struggling with this were asked, "How many steps are there between the freezing and boiling points?" - "180°F and 100°C." "Which then are the larger steps." - "°F."

46. D.A. Interconversion of units kWh ↔ Kcal ↔ Btu, etc. always presented difficulties for some.

47. There are still some students who have difficulty with the concepts of "up" and "down" in relation to a spherical earth. Are the Australians down?

48. Axis of sun and moon when rising and setting.

49. Averages. How can an average family own 1.7 cars?

50. Most of the class were troubled by the concept of the amount of solar energy intercepted by the earth being dependent on the section of the sphere and not half the surface area.

51. The blue of the sky and red of sunsets.

52. Sine waves.

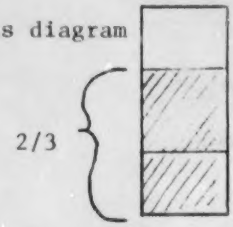
53. There was frequent difficulty with proportional problems of this nature: A wood stove burning at 66% efficiency produces 25,000 Btu's/hr. How much heat would it produce burning at 100% efficiency? (This would be incidental to some more complex problem.)

Many students argued along these lines:

66% - 2/3

So add 1/3 to make the total, 1/3 of 25,000

This diagram quickly convinced them to add ½ of 25,000



The Class as a Medium for our Research

While it became apparent during the first semester that this class was not ideal for our purposes, it seemed sufficiently fruitful for us to persist for another semester. More difficulties were uncovered, but the principal return was a very substantial confirmation of the difficulties that had already arisen. However, in the light of our experience of the teachers' courses discussed in Papers

it is now clear to us that these gave a much better return for the effort involved. The clear advantages to be gained from a small group of selected subjects, who quickly became completely at ease, who have no examination to face and so nothing to lose, and who have a professional interest in the process of learning, are described in that paper. Above all, the fact that we were exploring a topic rather offering a course meant that we were free to modify the style, content and pace of the class to suit the needs of individuals in a way that would have been quite inappropriate in a University course offered for credit which had to meet certain requirements of the College of Arts and Sciences.

1. Size of Class - Even the lab sections, groups of 25, were too large:

   a) for students to be willing to air their difficulties;

b) for instructors to give sufficient time to answer questions that were asked;

c) in spite of the very friendly atmosphere, for all students to develop full confidence in the lecturers or their classmates. While students were encouraged to make appointments to discuss difficulties individually, many of them did not accept this offer. Interestingly enough, at the mid-term exam in both semesters students were in severe difficulties but at the end of the course very few failed.

2. Pace of Class

In a normal class, the instructors have a responsibility to teach all students; however sympathetic and interested they are in those with difficulties, the body of students has certain expectations which one must try to meet. So one is forced to help the stragglers to overcome their difficulties as quickly and effectively as possible - one has to help them to jump hurdles, whereas in the teachers' courses we were able to spend perhaps several sessions digging deeper and deeper to find out the fundamental nature of those hurdles and how they had been built up.

Even in the lab/discussion sections, time for clearing up difficulties was too limited, because if too much time was spent on one set of difficulties these sessions would get behind the lectures. So the next set of problems were not being dealt with as they arose.

3. The Value of Participation in these Courses

In spite of these limitations, our involvement in these courses was useful because it gave us a broad survey of a range of problems, some expected, others still surprising. Only by continued observation of this kind, at all levels, can a catalogue be built up fo the barriers that exist, on the one hand to provide the raw material for more intensive study and on the other to guide curriculum developers.

Section J

NOTES AND REFLECTIONS ON COURSE

MAN AND NATURE: ENERGY

Maja Apelman

CONTENT

Introduction

Asking Questions and Needing Answers

Conditions for Learning: Trust and Motivation

Knowledge Assumed

Quantity of Material

Trying to Understand

Avoidance

Mathematics

Heat and Light: Some Early Struggles

J

Notes and Reflections on Course

Man and Nature: Energy

Introduction

The course Man and Nature: Energy was offered for the first time in the fall of 1980. It was intended as a course for non-science majors which fulfilled the natural science requirements of students in the University's College of Arts and Sciences. The course was taught by a team of instructors from three different disciplines -- physics, biology and agricultural botany. The course syllabus stated: "...Nothing quite like this has ever been taught before at the University of Colorado... We trust that you will enter into the spirit of the experiment by providing continual commentary on what you find useful and what you do not... Share your concerns with us. Let us know when you are confused and don't be afraid to say so. You may be the only one who will admit it but surely you are not the only one who is confused... Each of the instructors is interested not just in science, but in the teaching of science. We want to know how to teach better and what sorts of concepts and techniques pose difficulties for learners..."

I attended the class as a student and participant observer. Ron Colton, another member of the research staff, was one of the three insturctors. He lectured in the course and taught one of three lab/discussion sections. The other instructors were David Armstrong and Benno Klank.

The statements from the syllabus which encouraged students to voice their confusions were repeated in the first lecture class and in all three lab/discussion sections. It was made quite clear to the students from the very beginning that the three instructors were genuinely interested in the learning process, and in helping students overcome any difficulties they might have with the content and approach of the course.

My main function in the course was that of "student advocate": I was to be an uninhibited question-asker, with the hope that my questions would encourage the students to speak up freely when they were troubled or confused. I also expected to act as a liaison person between students and instructors; I know from experience that it takes a long time before students feel free to expose their ignorance with questions which they consider "dumb."

I encountered some initial problems with my role which made the early weeks of the course rather frustrating for me. These were, however, problems which could be prevented. Other problems were directly related to the fact that data was to be obtained from a regularly scheduled college course. It is difficult to uncover students' critical barriers in a course in which a certain amount of material has to be covered. The instructors were often concerned about falling behind in their schedule, while many students were still having difficulties with some of the course material, especially where mathematical calculations were involved. Peter, a senior in political science who had successfully completed a calculus course, commented to me how little material had been covered a month or so after the beginning of the class. "They could have covered a lot more," he said, "but then we wouldn't have understood anything."

One possible remedy for this situation in the future could be the addition of another lab/discussion section for which students would volunteer, for additional credit, and in which difficulties could be dealt with at greater leisure. In such a class, students could also be asked to keep a record of their learning problems, thus enriching the data for the research. Although many students found aspects of the course quite difficult, others probably felt that the course was moving too slowly.

I loved going to this class. I was interested in the content, took notes at lectures and lab sessions, did most of the written homework, and even took some of the tests (though on a take-home basis). I learned a great deal and was excited by my new understandings.

The following comments are based on my class notes, my personal reflections written during the time of the course, and on questions asked by students (including myself) about some of the material presented.

Asking Questions and Needing Answers

"Are there any questions?" is routinely asked by most teachers when they are presenting new material to a class. More often than not, no questions are asked and the lecture proceeds.

It took a while for students in the energy class to feel free to respond to this question, though as the course progressed questions became more frequent and were asked more spontaneously. Questions asked in the large lectures were mostly factual. It was clear to everyone that lectures were not the place to deal with deeper confusions. Yet even in the smaller discussion groups, not every student asked questions. Some did not need to ask. Others, however, held back. Since it was part of my job to encourage students to ask questions, I watched carefully for signs of confusion. Quite often I noticed students who looked puzzled but did not voice their concerns.

In critical barrier research, questions provide the most valuable information on students' thinking and understanding. Yet the structure of a college class presents some real difficulties. Class size, the amount of content to be covered, the traditional lecture style of most college teachers, all would have to be modified, and time greatly expanded for questions and answers. From my own experience, I know that it is hard even in the best classroom situation to ask all the questions you may have. You begin to feel that you are monopolizing the class, boring other students, and possibly exasperating the instructor. You also become very sensitive to instructors' expectations, whether real or imagined. When the confusion is deep and explanations have not helped, there comes a point when you say "I understand" just to get out of a painful situation.

Feeling free to ask questions is one aspect of question-asking. Another aspect is needing an answer. When learning something new, there is an early stage of confusion when it is difficult to even formulate a question. With time, however, the learner comes closer to understanding. Question then suddenly becomes urgent.

On November 3, 1980, I wrote in my notes:

> When I first began to wonder whether heat always flows to something colder I needed to know right then whether this was so. Benno was gone but luckily Ron was there and I could ask him. He said yes, that is more or less how it works.
>
> Why do I have this sense of urgency? Is it that I'm afraid I'll forget? Or is it that it is the end of a long road, some of it not even conscious, and when the goal is finally in sight I want to get there fast to resolve the tension? I wonder if this is what children feel when they are learning? Is that why some children ask questions so incessantly? I can really empathize with that now: I had the same feeling when I was talking with David A. last week. I was onto some understanding and he had the answers and I wanted to get them. The only problem is that often an answer raises more questions and you can stay and stay and ask and ask until you become utterly exhausted. I really wonder whether that is why curious children never stop asking questions!

Dealing with students' questions, encouraging them, answering them and providing experiences that will help students to find their own answers -- these are central to critical barrier research. It is a slow, time-consuming process, not well suited to a college course which has to meet other obligations.

Conditions for Learning: Trust and Motivation

A visitor to one of the lectures in the energy class remarked that students were led through many calculations to an end result which they did not understand. The visitor thought it was a bit as though the instructor were the Pied Piper saying, "Follow me, and we'll get to some nice place." (The calculations were about converting watts to calories to footpounds. It was a rather belated summing-up of experiments done by groups of students earlier in the course.) I took notes during this class but did not really try to understand the calculations, I wrote in my notebook, "You have to have a reason to want to do all these conversions." For most students, the reason was the upcoming midterm exam. I did not have to take the test, so I did not make the effort to follow the calculations.

The visitor did not know that we had dealt with the topic on many previous occasions, but his comment made me think of how much a student has to trust the instructor when the direction of a lecture or the purpose of an experiment is not immediately apparent.

> Benno likes to have things going on - experiments during a lecture - without telling students what it is all about. "Just remember it, we'll get back to it", he says. If you know your teacher well and trust

> him or her, you can accept this open-endedness and not get too concerned. For some students, however, this style of teaching was difficult to accept.

There is a lot of open-endedness in science, which comes as a surprise to the nonscientist. During a lab session on light and heat, I wrote in my notebook: "Does everything have to lead to other, more complicated things?" It took me a long time to become comfortable with this lack of closure. The more I know about a subject, the better I can accept open-endedness -- probably because my existing knowledge gives me the security to be left hanging. But when I delve into new subject matter, my tolerance for open-endedness decreases noticeably. I feel insecure with my lack of knowledge and I push hard to gain some understanding. At that stage, I am not happy with postponements of explanations.

> I wanted to know how light and heat were related. Benno said "I have to get into atoms and molecules to explain that." I was irritated and thought: "I don't care what he has to do; I don't want to keep being confused and I don't want any more holding answers." To me light and heat are not the same. Tara obviously was not connecting them either since she wondered: "Why can't you make a bulb that makes light without heat?"

How can students be helped to accept open-endedness, ambiguity, and temporary confusion as a normal part of the learning process, and how can they be better prepared to tolerate the discomfort that often goes along with not understanding?

Being personally motivated to understand something can be most helpful. I had been confused about amps, volts, watts, and ohms for years, and I never did sort it all out during the energy course. But the other day, when an electrician was wiring an addition to my house and I wanted to make sure that there would be enough power to use an electric oven, I was motivated to overcome my block. I was talking watts to the electrician; he answered me in amps. Right there I decided the time had come for me to get this confusion straightened out. I asked Ron for help, and this time I understood. Now I have trouble remembering what my problem was and why it took me so long to understand! It all seems rather simple and straightforward.

Knowledge Assumed

There can be quite a gap between an instructor's idea of a simple explanation and a student's ability to grasp that explanation. Here is an example of Ron trying to explain shadows to me:

> You know that the sun's rays are perfectly straight;
> (I never thought about it.)
> But because the sun is so large, they cross...
> (Where? I forgot. Is that the focal point?)
> ...and make a little triangle...
> (I tried to draw that but couldn't get it right.)
> ...and then with some simple geometry you can calculate the distance to the sun."
> I must have looked puzzled, so Ron tried again:
>
> "You know that if the rays of the sun were straight you would get a very sharp shadow all the way."
> I never thought about the sun's rays being straight or at an angle* much less about a shadow being related to the sun's rays, and I would never wonder why the shadow is sharp at one end and fuzzy at the other - in fact, I've never even noticed that. There is so much knowledge assumed that I don't have that I can get quite overwhelmed.

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*Last year I was thinking "straight" vs. "at an angle." Now I know it is "straight" vs. "curved."

Assuming knowledge that a student doesn't have is an almost universal problem in science teaching. It is reasonable for college instructors to assume some basic knowledge, especially in subjects like mathematics and biology which are required in most high schools. But if the students don't have the knowledge, and come to a college course for "non-science majors" with a lot of fears about their ability to understand science and perform mathematical calculations, how are the instructors to proceed? What is the solution to this dilemma?

Quantity of Material

When does a learner reach a point of intellectual saturation? This is an individual matter; it will vary from student to student and it will vary for the same student from day to day. External factors such as fatigue, hunger, preoccupation with personal problems obviously will influence how open a learner is to absorb new information. But even under optimum conditions, well-rested, well-fed and initially relaxed students often reach a point where they don't want to or cannot take in any more material. I can give many examples where I reached this point of saturation. One lecture stands out in my memory as being overwhelming not so much because of conceptual (barrier) difficulties but because of the sheer quantity of material presented. I wrote in my notes:

> That was a frustrating lecture for me, especially at the end when we got to the carbon cycle. There's a lot I don't know and there's an awful lot of new material being presented in class right now. The water cycle was all right, but the carbon cycle?

> First of all I got confused because, in the previous labs, we were burning coal and that was called carbon. So when David first mentioned the carbon cycle, I had this little piece of coal in mind...OK, I learned that carbon moves as carbon dioxide and I sort of knew that was a gas, but carbon dioxide gives me problems. I never realized air was not just oxygen and I have a hard time fitting carbon into my mental image of air. I've also heard more about carbon monoxide - which is poisonous - and I don't see why one more oxygen molecule can make such a difference.
>
> I can't handle new material when I don't understand most of it and I get anxious if I'm told too many times not to worry about it because "it will be covered later." Like photosyntheses. The carbon dioxide combines with water (where? in the leaf, in the air?) to make carbohydrate. I never realized that carbohydrate means carbon and water. Then it is burned. What is burned? Fossil fuel which was made of carbon? How? And then there was a picture of a cow labeled "Respiration" on a chart illustrating the carbon cycle. Respiration seems to be what happens after animals and human being eat. My own association was with breathing but I was told that respiration is a parallel process to combustion and at the end of the cycle, everything goes back to CO_2. I am really confused! 9/29

There was a whole series of packed biology lectures in which the quantity of material presented was too much for me to absorb. However, "biology overload" never affected me in the same way as too much physics or math.

> Even though I had a hard time following what was presented -- mostly names of different parts of a cell -- it was not anything that I felt I could not understand with time. I never got that panicky feeling or the anxiety which can overcome me when I don't understand the math or can't grasp the physics.
>
> The photosynthesis process -- the work of the chlorophyl molecule which splits the water molecule with the help of sunlight -- may be mysterious but it does not seem to me difficult in the way that some of the physics concepts are difficult. I don't know how you would experiment with it to find out what really happens, and how the hydrogen that is

> split off from water combines with the carbon dioxide to make glucose, but I trust it can be done in a lab. 10/10
>
> I wish I had more time to absorb things. Why don't I get more excited by biology? David is excited by the fact that the mitochondria and chloroplasts run the biosphere but that is pretty abstract to me. However, I know now what I will have to understand in order to understand photosynthesis. Until now it has been a complete mystery. I have a vague understanding of the role of chlorophyl and I also know that sunlight is the key in this whole process. I think with time to wonder and reflect and a few more explanations, or some simple books, I could get it.
>
> The discussion of the rain forest was interesting: there is no growth at the bottom because there is no light. Philodendrons have roots in the ground but they have to wind around a tree up to a height of 300 feet to reach the light. I've always known that plants turn towards the light or reach up to the light to grow but I didn't know they have to have sunlight to live and that "living" for plants is making carbohydrate. 10/24

There were times when I experienced another kind of overload situation. When I asked for help or information, I often was told more than I wanted to know at that moment.

> I could have done without the introduction of latent heat today. I just wanted to dwell on my very exciting understanding of heat flow and have lots and lots of examples to make sure I really understood it. 11/3
>
> I was checking out my understanding of solar heat with Benno, having just realized why you have to take into consideration the amount of time an area is exposed to the sun. (You use up wood or coal when you burn it but you don't use up the sun - therefore you must time it.) Benno said it was similar to electricity but I didn't want to hear anything else just then. 10/10
>
> Yesterday, when I had figured out that heat flows to cold to sort of even things out, I was very excited and I wanted to enjoy my little triumph. A day or two later I would have been ready for more. But Ron

> told me right away about entropy. I didn't like the idea of entropy - I didn't want anything so awful sounding mixed up with my new discovery, and I really didn't want to hear about anything else right then. I just wanted to enjoy what I understood, savour it and get it firm in my mind before going swimming again in a sea of new information where I would have to work hard not to drown. 11/14

Teachers tend to give students too much information - more than they have been asked for. At times, such new facts or ideas become stimuli that further the students' understanding and make them aware of new possibilities or directions. At other times, however, students may need more time to digest, integrate or simply enjoy their understanding, or they may want to try it out in new situations until it becomes firmly rooted. They are not ready, just yet, to move on. In their enthusiasm to help, teachers may rob students of the pleasure of having reached a new level of comprehension.

Trying to Understand

All through my notes there are comments about being exhausted by the effort I had to make to understand something or frustrated when I didn't get it.

> The demonstration of the bomb caloriemeter and all the ensuing calculations were almost too much for me. Benno is a good explainer, very patient, but my ignorance is great and the effort I had to put into understanding everything got me exhausted. 10/2

> It is hard to be so bombarded with new facts and concepts and not have time to understand them. It is exhausting. 10/4

> I concentrated really hard today and followed along almost all the time. This concentration is draining. 10/7

> (After seeing a film on astronomy - The Nearest Star) What really are waves? and what is electromagnetic radiation? What radiates? light? heat? waves? And what's the electromagnetic field of the earth? I get so frustrated when I understand so little or when explanations are full of terms I don't understand. 9/26

I wish I knew whether the students in class felt a similar frustration or exhaustion. I doubt that they did. I'm sure many of them didn't understand things any better, but they probably didn't try as hard as I did. Perhaps they did not expect to understand everything or had learned not to mind. When do students begin to accept "not understanding" as a normal part of school?

On November 3, I wrote in my notes:

> Over and over again I am struck by two things in my efforts to learn science:
>
> 1. How difficult it is, what an incredible mental effort I have to make, to follow the lectures and lab/discussions. I can feel utterly drained and exhausted, even hungry, after only one or two hours of classwork.
>
> 2. How frustrated I become when I don't understand something and how excited I can get when I do gain a new understanding.
>
> I wonder if science teachers are aware of the extent of frustration which many students experience. "What *is* it that I am not getting?... *Why* can't I understand this?"... "I don't really know what I'm trying to understand anymore!" Such feelings can be very anxiety arousing. I suspect that children react this way in school from the earlierst grades on.

Avoidance

There were times when I made a conscious decision to back off, when I simply did not want to or could not make the effort to understand any more.

> ..."There are electromagnetic waves all around"... I am tired and it takes too much of an effort to understand this.
>
> After a lecture in which I finally began to understand how light is changed to heat -- light hits something, is absorbed, some of it radiates back as heat on a different wavelength -- Benno went on to a discussion about why the sky was blue and the sunset was red but I didn't even try to understand that. Another time.
>
> We've gotten into radiant energy in class and that gets us right smack into light waves -- a subject I have avoided for many years. Although I knew that I would have to tackle it one day, because I often got into topics which required that understanding, I was never quite ready to make the effort. The reversed image in the pinhole camera, the round sunspots in the shadow under the large elm tree, the colors in a prism; rainbows -- these and many other experiences touched on the subject of light. Yet whenever I looked at the sun, a lamp or whatever -- I would retreat. "Some day I will have to learn it", I said to myself, "but I'm not up to it right now."
>
> Now I HAVE to learn it. And I'm probably more ready than before since I have at least some awareness of light rays. They aren't a complete shocking sort of surprise, forcing me to rearrange comfortable existing knowledge. My "naive" understanding has been jolted many times before. 11/11

I wish I could describe the feeling when I don't understand something and am trying to decide whether to make the effort to struggle with it and work it through. As I said earlier, this decision is generally influenced by such outside factors as fatique, preoccupation with personal matters, lack of interest or the amount of tolerance for possible frustration you may have on the particular day. Any of these reasons may contribute to your decision not to make the mental effort to follow a new idea or struggle with a new concept. At other times, however, the decision to pull back is primarily due to the fact that you have reached a point in your thinking where you know that "sticking with it" would not be fruitful. Sometimes it feels almost like a kind of rebellion -- "I've had it for today, I'm not going to make the effort, it's just too hard." At other times it is simply a matter of too much material to absorb, or it may be a topic with which you have struggled unsuccessfully before and you've decided that you won't get it anyway, so why even try. It is always a conscious decision that you make -- "not now, perhaps some other time when I'm more ready for this." I believe this is a defense against feeling overwhelmed by lack of understanding.

Mathematics

I have left the description of mathematical problems to the end. Just looking through my notes and trying to reconstruct the thinking behind some of my calculations brought back all the anxiety I felt last year when I attended the class. "I don't understand this anymore...I don't want to go through these calculations again...I still don't understand why you divide grams into calories...". Those were some of my thoughts as I looked at my notes. In class, I almost always got lost when calculations were done on the blackboard. When I managed to follow them, I usually could not recall the logic of the calculations later on. Things went too fast for me. I copied everything into my notebook and then tried, after class, to reconstruct the calculations. Sometimes I could figure it out;

sometimes I needed help; sometimes I got the idea but didn't want to spend the time to work it through. It never seemed to get easier, and the same type of problem kept throwing me off. The problem was simple division! It was not how you do it, but why you divide to get a certain answer. Here are some notes illustrating how I was struggling with division:

> Ron took a globe about one foot in diameter and asked how far away the sun would be on that scale. He told us that the diameter of the earth was about 8,000 miles and the distance of the sun from the earth was about 93 million miles. He then divided 93 million by 10,000 -- an order of magnitude calculation -- to get an answer of 9,300 feet. That's how far the sun would be from our one foot globe.
>
> I had to repeat the calculation with my method - learned, I think, in my high school algebra class. I had no idea why Ron was dividing the distance by the diameter.
>
> 1 foot is to 8,000 miles as X is to 93,000,000 miles
>
> $$1 : 8{,}000 = X : 93{,}000{,}000$$
> $$8{,}000X = 93{,}000{,}000$$
> $$X = \frac{93{,}000{,}000}{8{,}000}$$
>
> I can set up the problem and I know how to solve it but I don't really understand why you divide or why the answer is in feet.
>
> A similar division problem came up in Benno's class:
>
> If .6025 grams of a peanut butter sandwish -- the actual weight of the sandwich pellet we used in our experiment -- produces 2319.5 calories, how many calories are produced by one gram? Benno divided 2319.5 by .6025 to get an answer of 3849.8 calories. Again I asked: why do you divide .6025 into 2319.5 to get the number of calories in one gram? It just tells you how many times .6025 fits into 2319.5. What does that have to do with one gram?
>
> Since I have trouble understanding this, I obviously, never understood division.

Benno said division is how many times something fits into something else. That still doesn't quite explain the above calculation.

I do get it, though, with simple numbers:

If 2 grams produce 4 calories
1 gram produces 2 calories

No - it doesn't help, I'm just taking half of each!

I became so frustrated and irritated with myself for not understanding why you divide that I spent considerable time trying to figure it out. I think I came to understand it at the time, but reading over my own notes, I am still a bit confused.

A problem which involved leaving a term out of an equation when its value was 1 came up in Benno's class.

We had exploded a small pellet of peanut butter sandwich in the bomb calorimeter and were trying to figure out how many calories were contained in the pellet. We had previously learned that one calorie raises one gram of water by one degree Fahrenheit. Today the term "specific heat" was introduced. When the sandwich pellet exploded, it heated the water that surrounded the container, and the water, in turn, heated the container. In order to calculate how much heat the sandwich pellet produced we had to calculate: 1) How much energy (calories) went into heating the water - the amount of water multiplied by the temperature difference - and 2) How much energy went into heating the metal.

We now were told about specific heat and given a formula

mass x specific heat x temperature difference = calories

Then Benno mentioned that the specific heat of water was 1, the highest of any material known, and the specific heat of iron/nickel (the metal of the container) was .1. We now plugged in our figures:

> 3703 g (nickel) x .1 (sp.ht.) x 1.05 (temp.diff.)=388.5 calories
>
> 1838.8 g (water)x1(sp.ht.) x 1.05 (temp.diff.) = 1931 calories
> Total: 2319.5 calories
>
> Since we only had .6025 grams of a sandwich (not 1 gram) we must divide: $\frac{2319.5}{.6025}$ = 3849.8 calories.
> There were 3.85 calories in that little pellet!
>
> This calculation got very confusing because the specific heat of water had not been mentioned before. We had been calculating calories by using only the amount of water and the temperature difference. It had worked because the specific heat of water was one but we were never told that. Now we suddenly had to multiply the nickel by the temperature difference as well as by .1 and this took us a while to understand.

Throughout the course, simple math problems became major obstacles, for me as well as for many of the students. The same kind of elementary arithmetic problems kept cropping up time and again. When do you multiply? When do you divide? Why do you multiply or divide? Formulas were confusing, however simple they appeared to be. For instance: one calorie raises the temperature of one gram of water by one degree:

> A student in Benno's lab wanted to know why you multiply the degrees raised by the quantity of water to get number of calories. I understood it but couldn't explain it to her until I suddently realized that the one calorie was the result of multiplying one gram of water by one degree. We just accept formulas and definitions but we don't know how to apply them because we don't really understand how they are arrived at.

Simple math problems interfered constantly with my science learning. When you get hung up on division or multiplication, you can't begin to concentrate on the science being taught. You are stuck, become frustrated, and tend to panic or tune out.

It is a humiliating experience when you start to think:

> I'm not getting what others are getting... I'm not getting what the teacher thinks I should be getting...I thought I understood multiplication and division -- I've used them all my life so why can't I understand it now? Every time a calculation is done on the blackboard, I wonder whether I'll be able to follow it...I'm not having trouble with the science concepts, it's the damn arithmetic that's getting me down!

As part of an order of magnitude calculation -- figuring out approximately how many rice grains there would be on a chess board if you doubled the number of grains on each successive square -- we were trying to calculate the number of grains of rice in one cubic inch. Benno was doing it on the blackbaord during a lecture. We estimated that one rice grain would measure approximately 1/16" x 1/16" x 1/4". How many rice grains would fit into one cubic inch? A student called out "16x16x4." I was amazed how anyone could come up with an answer so fast. Benno did the calculations slowly and carefully, since many students didn't understand it, but I still had to work it out for myself after class, making many drawings of a cubic inch and rice grains.

Here a good example of a teacher trying to make things simple:

Excerpts from Explanation of Order of Magnitude Calculations

> Benno lab, 9/15/80
>
> Benno: Let me show you an example of an order of magnitude calculation. The first thing that we will do is try to figure out how many grains of rice there are in a cubic inch. Okay?

So, have a guess, how many are in a cubic inch?

Student: About a hundred.

Benno: A hundred...four hundred...a thousand (writes on board). Fifty. Do you think we have a range which is reasonable?

Laughter.

Student: Thirty-five

Benno: Thirty-five. Okay, talking about order of magnitude calculations, thirty-five and fifty is the same. You know we are looking at such a broad spectrum that thirty-five and fifty fall into the same range of ball park figures, they should be in the same order of magnitude. Obviously nine hundred and one thousand would be the same for this purpose here. We are not interested in the last little grain of rice, we are just trying to get some kind of an idea whether there are maybe 10, 100, a thousand or ten thousand in there. All right, the reason is of course that in the end we want to figure out how much rice there is on that chess board. And again you see, there it will not matter whether we are off by one or two grains, or whether we are off by one or two cubic inches. So let's do it. How big would you say is a grain of rice?

Student: A eighth of an inch long.

Benno: A grain of rice looks pretty much elongated, you know a little bit like a fat hot dog or something like that. What would you say is the diameter of the thing? A sixteenth? A sixteenth I think is close. An eighth of an inch is too big if you think about it. It's way smaller than that. A sixteenth of an inch maybe, in that neighborhood. Okay, how long is it?

Students: A quarter of an inch.

Benno: A quarter of an inch. Okay. Now we'll make life easy for ourselves. Since we're just interested in the order of magnitude calculation we will assume that all rice grains look like this. Then we can stack them more easily or we can calculate the stacking of them more easily. Okay, grains of rice are one quarter on an inch long and they are one sixteenth by one sixteenth of an inch. Good, we've nearly got the problem solved. How many of these grains of rice go into a cubic inch?

Student: Sixteen, times sixteen times four.

Benno: Sixteen times sixteen times four. Does everybody see how we got that?

Student: NO!

Benno: All right, that is a cubic inch, right? Then let's take one grain of rice. How much is the area that we have here?

Student: A sixteenth of an inch.

Benno: Okay, if I take sixteen of those little areas, I will get a length which is one inch long, but does it fill the entire area of one inch by one inch?

Students: NO.

Benno: No, so sixteen of those little pieces here simply make one narrow string which is one inch long here. How many of those little strings do I need to fill this entire face? Sixteen, okay. So I have sixteen here and I have sixteen here, right? In other words, this little piece here which is a sixteenth of an inch on each side, that little piece is what fraction of a square inch? One sixteenth of a sixteenth, right? What is that?

Student: One over sixteen squared.

Benno: Sure. Sixteen squared...sixteen times sixteen is 256. Okay, so I need 256 of those dumb little grains of rice ot make one layer of rice which is how thick now? One fourth of an inch. So how many of those layers do I need to make up a complete cubic inch? Four, right? So I need four times 256 grains of rice to fill one cubic inch, so what is that?

Student: One thousand.

Benno: That's one thousand, right. Okay, because you know that we are off by a few kernels one way or another. One thousand are in one cubic

> inch. Who believes that? Who does not believe that? Does it sound reasonable? I had one student who just didn't believe it. This is kind of old and it has been bent, but you can get the idea. She made herself a little cubic inch, and she filled it with rice. I still have the rice here. She counted it, and she got nine hundred and eighty nine. Do you want to recount them? There they are. Do you see now what is meant by an order of magnitude calculation? You make some reasonable assumptions, like rice is about a sixteenth of an inch thick and a quarter of an inch long.
>
> Student: How close do you have to be?
>
> Benno: That depends on the problem. Quite often it's good enough if you are even within a factor of ten....
>
> (discussion now focuses on order of magnitude)

Large numbers were troublesome. I had no familiarity with them, and no facility to make quick calculations. I remember even wondering how 10 x 10 x 10 could be 1000. I had no problem with 10 x 10 being 100 or even 10 x 100 being 1000 but for some reason 10 x 10 x 10 didn't look large enough to get to 1000 so fast. I don't really know why I thought that way. I probably just looked at the numbers, all three of them quite small, and was startled when the answer turned out to be so large. Perhaps the number 10 was the thing I concentrated on: 10x10=100 and then you multiply "it" by another 10. The "it" is of course 100 but somehow you think about 10 and then wonder where the 1000 comes from! How could any college science teacher really believe that students could get hung up on 10x10x10?

The problem is vast and while I don't know the solution, I can make two simple suggestions. The first concerns the structure of the energy course, or other similar courses aimed at non-science majors. If simple math turns out to be such a major problem, why not offer a short math review course at the beginning of the term -- as soon as the first problems arise -- so students have a chance to gain the understanding necessary to apply this basic arithmetic? I realize that such knowledge should be taken for granted in college, but if it doesn't in fact exist, the students who are already timid about math and science will only become more apprehensive as they experience constant frustrations with relatively simple calculations.

The second suggestion is broader: much has been said and written about math anxiety, and many courses exist at various universities aimed at helping students overcome this anxiety. I wonder, though, if sufficient efforts are being made to document the student's thinking and feelings while they are in the process of tackling a problem. I believe it would be most valuable to find out how students are thinking when they don't understand something, and when and why they get confused. Just knowing that students get anxious does not throw sufficient light on the barriers that interfere with their learning.

In the concluding section of this essay I will present one attempt at documenting how a scientifically naive student (me) tried to master some of the basic understanding of heat and light.

Heat and Light: Some Early Struggles

I have started many times to keep track of my thinking and learning when I am trying to understand a difficult new idea. I enjoy writing things down, and find that my thinking becomes more focused as unexpected questions and answers pop into my mind. I have, however, found that it is extremely difficult to keep track of all the thoughts that relate to the learning of a new concept. I would virtually have to walk around with a tape recorder so I could talk into it whenever I have an insight, a question, or a confusion. When I'm really trying to understand something that gives me trouble, thoughts come into my mind at quite unpredictable times. I might be driving, or relaxing in the tub, swimming, cooking, or talking to a friend. Although at the time I'm always sure I will remember my thought until I can write it down, most often I forget it. I get ideas when I think about the topic, but also when I'm not thinking about it. I may be walking down the street looking at a familiar scene and suddenly I see things in a different light because I have absorbed new knowledge. (As I was writing this sentence the word light made me think of waves and the word absorb made me think of heat -- both new associations.) It is almost impossible to catch the whole progression, but it is crucial to keep as many notes as possible along the way: when I reach new understanding or gain an important insight, I cannot reconstruct how I was thinking before. Even if I remember how I used to think, I can no longer relate to it. The old way has been replaced by a different logic and understanding and it seems strangely out of place.

Following are some notes I wrote during the first two months of the energy course. They are mostly questions I had about the properties of light and heat. The notes are sketchy but I believe they catch at least some of my early thinking and confusions.

September 15

Benno had a contraption in class today which illustrated that different metals conduct heat at different rates. Little metal balls were stuck onto the end of metal rods with wax. When the rods had conducted the heat from a central source to the balls, the wax melted and the balls fell off. There were two rods made of iron, one thin and one fat. I don't understand why the thick iron rod got hotter faster than the thin one.

September 30

I really enjoyed going out with the little solar collector. This whole big confusing subject -- solar energy -- suddenly became quite graspable. Water is heated by the sun. A black material behind the water absorbs light, which becomes heat (I don't understand that yet) and insulation behind the water keeps the heat from escaping, so the water gets warm. So far so good; I did have some questions, however:

What is the relationship between heat and light? Are they the same? Does light go through the glass, get trapped behind it, change to heat and stay hot because water is a good heat storer?

Greenhouse effect -- locked cars in the summer: Why do I have to think of light as being changed to heat inside a car? Why can't I just think of the heat from the hot sun as getting into the car and because the windows are closed it stays in there? I never really wondered before how the heat gets into the car: everything that is closed and stands in the sun gets hot! What does absorb really mean? Is it the opposite of reflect?

October 6

I think of the cold air coming into the house, not of the warm air flowing out. After all, you feel cold draughts when you sit near a window. Why do you feel that if heat is flowing out?

Heat, it seems, always flows to where it is cooler. So if it is cooler outside, the warm house air flows to the cool outside. Strange. Why can't it just stay in the house? Does heat always have to move?

Benno tried to explain this with an analogy: a bucket of water has leaks. You can stop the water from escaping by plugging up the holes or by constantly adding water. So with heat loss in a house: insulate or heat a lot. But I see water flowing out of a bucket; I don't see or even feel heat flowing out.

October 7

Heat is a weird thing. It flows. It flows to where it is colder. Does that mean there is no cold? Only more or less heat? Does heat sort of want to equalize things? Does it want to make the surrounding cold air or substance warmer so it gives it enough heat till the two are mixed equally?

I got onto some general questions in my thinking: A substance (like water or nickel) has a certain capacity to absorb heat; to retain or store that heat; to heat surrounding areas, and to transmit (conduct) heat. All these are obviously related. Heat is absorbed at different rates by different metals (remember experiment) and transmitted (conducted) at different rates. I sort of knew that. But I never thought before about heat being stored!

Why do the water and the container it is in have the same temperature? Is that always true that heat flows out and tries to make what it comes in contact with warmer?

October 8

Summary of my present understanding of heat:

(I should have started earlier to write this down. I am already thinking differently about heat. Or rather - I never thought about heat before except when it was too hot or too cold for my comfort.)

Heat radiates (from the sun).

Heat is related to light (from the sun).

Heat is reflected - or is it? Light is, anyway.

Heat (and light?) are absorbed.

Heat gets stored in substances.

Heat travels through substances.

There are ways to measure how much energy (calories) are needed to heat up a substance.

Heat likes to warm up what's around it to its own temperature.

Light goes through glass and gets trapped and changes to heat. (Greenhouse effect.)

That's all, so far.

October 31

I missed two lectures and I haven't read the textbook because it seems utterly inaccesible, but something is beginning to happen in my understanding of heat and light.

They are not the same but they are related. Benno says it's like dollars and steak: if I have dollars, I can buy steak with it; if I have a steak, I can get dollars with it. But when and why and how are they exchanged? When does light become heat, and heat become light? That is not yet clear to me.

Then there is absorbtion and reflection. They are opposites though it is hard to understand how something you can't see gets absorbed. I can see light being reflected; I can't see light being absorbed. The substance that absorbs it doesn't get lighter; it gets warmer -- black? There is transparency - light can go through some things but not through everything; and different wavelengths of light go through different things. Which gets me to the spectrum and wavelengths and frequencies.

Somewhere in there is flow: energy flow, heat flow. Do they talk of light flowing too or only heat?

November 3

I asked Benno to explain how light and heat are related and he started to talk about electromagnetic waves -- they are all more or less the same in character, they differ only in wavelength and in intensity. (I'm not sure what that means.) Everything continually emits and absorbs waves. Benno said he makes no difference between light waves, radio waves, etc. (What does that mean? Do I emit waves too?)

Energy goes along with radiation. How does that work? Is everything that is warm, that has some temperature above absolute zero, emitting radiation which is waves and also heat? To understand light and heat being the same, you have to understand that it's all waves.

Benno asked at one point: how can I make light? By heating something. But - even before it gets hot and burns and gives light, it emits waves. It has some heat in it. The wood or coal that is burned has heat in it. That puzzled a lot of us. That's when Benno switched to the Kelvin temperature scale.

Now I think I am beginning to understand what it means that there is no cold, only an absence of heat. Everything has a certain amount of heat, from absolute zero to the inside of the sun and all temperatures in between. Heat flow must mean, then, that when two things of different temperatures come in contact with each other, the heat flows from the warmer to the colder.

That wouldn't really be so terribly complicated to understand if we hadn't spent our whole life thinking that things are either cold or warm. My hand melts the ice because heat from my hand flows into the ice? In the process, my hand gets cold: not from the cold ice, but because it is losing heat to the ice - is that how it works?

Everything in our experience that we call hot is warmer than our body temperature. It seems that we use our body temperature as the norm to determine heat and cold. It is very strange therefore to suddenly think that when my feet or hands are really cold, they emit heat waves! Unless I redefine the meaning of heat completely, which I am just in the process of doing.

November 3 (evening)

I was trapped today. Last Friday I managed to leave the physics book alone and turn to other work I had to do. Today after dinner I was just going to look at the physics book for a few minutes while drinking my coffee. And now it is two hours later. And I am a bit closer to understanding. I'm learning to read the book without getting hung up on not being able to follow the formula stuff. Whatever I don't get, I ignore. But the section on how heat is transferred was easy; the explanation of temperature and thermometers was clear, and I even got a first approximation feeling about volume and pressure and constants and ideal gasses and density and all kinds of other strange things. I must be on some sort of plateau because I didn't panic. Getting the idea of heat, heatloss, heat flow, absence of heat was exciting and spurred me on. Now I'm on the way to understanding "waves" - electromagnetic ones. But when I was trying to explain all this to my son it didn't work too well.

"You mean what we see, the light, is all waves?"

"Yes"

"Well if the waves are all around, why can't we see in the dark?" That stumped me. Do lightwaves need light to be seen? I don't really get it. I am trying to live with this picture of waves coming down from outer space. What exactly is the role of the sun? Does it give us only heat, and light and other things emit the other kinds of waves?

This was my last entry on this subject. The following semester I was participant observer in another course -- a seminar for elementary school teachers which met one afternoon a week. One of the topics of that seminar was heat, though we approached it rather differently. Electromagnetic radiation was largely ignored.

In another teachers' seminar the subject under study was light and color. There we did discuss light waves and now I realize how little I understood when I wrote the above notes. Waves, however, still elude me. They seem to become more complicated the closer I get to understanding them.

After a year of struggling with science, I have learned at last that things won't get any easier. I know so little and every new understanding opens up areas that I never even thought about before. I have to learn now to live more comfortably with my limited knowledge and to enjoy the little victories along the way.

Section K

APPENDIX A

CONTENT

Bibliography

A Survey of Some Relevant Psychological Literature
by Connie Eppich

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A SURVEY OF SOME
RELEVANT PSYCHOLOGICAL LITERATURE

Connie Eppich

Paul R. Christensen, J.P. Guilford, and R.C. Wilson
Relations of creative responses to working time and instructions
Journal of Experimental Psychology, 1957, 53, 82-88

Found a relatively constant rate with time of responses produced for inventive or creative tasks. Found that uncommonness and remoteness of responses increased with time while cleverness of responses remained constant. Instructions to be clever decreased total number of responses but increased number of clever responses and average degree of cleverness.

Robert M. Gagne
The acquisition of knowledge
Psychological Review, 1962, 355-365

Asks the question: What kind of capabilities would an individual have to possess if he were able to perform a given task successfully, were he to be given only instructions? Suggests that the answer is the ability to perform a task which is in some sense simpler and in some sense more general. One can construct a hierarchy of these simpler, more general tasks which lead to an internal "disposition" to be able to perform successfully the task in question.

Identifies two independent variables that must be accounted for in a theory of productive learning: instructions and subordinate capabilities. One must also take account of their interaction.

Characteristics of instructions--

1. Make it possible for learner to identify the required terminal performance; define the goal.
2. Bring about proper identifications of elements of the stimulus set.
3. Establish high recallability of learning sets.
4. Promote application of learning sets to performance of unique task ("guidance of thinking").

Gagne suggests that learning sets are mediators of positive transfer from lower-level learning sets to higher-level tasks. He took the problem of finding formulas for the sum of n terms in a number series as an example, and broke it down into its component parts, or the various subordinate learning sets necessary to master the problem. He emphasizes the importance of individual differences in the capabilities people bring with them to a given task. For example, some must learn more of the subordinate learning sets than others before successfully completing the task.

Karl F. Neumann and John W. Barton
Factor analysis of a system of students' learning styles
Perceptual and Motor Skills, 1979, 48, 723-728.

The authors attempt to see if Rosenberg's four learning styles exist in a group of 377 eighth-grade students and find that this classification system does appear to have psychological reality. The four learning styles are rigid-inhibited, undisceiplined, acceptance-anxious, and creative. This approach may have a certain utility for explaining why some children learn science more easily than others, and it may be possible to tailor a teaching program to the child's individual style.

Jeffrey J. Flexibrod and K. Daniel O'Leary
Self-determination of academic standards by children: Toward freedom from external control
Journal of Educational Psychology, 1974, 66, 845-850.

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Compared the performance of children who were allowed to choose their own reinforcements for correct responses to arithmetic problems with the performance of children who had reinforcements imposed by the instructor, and with children who were given no reinforcement. There was no difference between the two reinforcement conditions while the no-reinforcement children had significantly fewer correct responses than did the reinforcement children. The children in the first condition tended to choose lenient reinforcement contingencies.

John R. Suler
Primary process thinking and creativity
Psychological Bulletin, 1980, 88, 144-165.

I enjoyed this article and found it to be, perhaps, the most intelligent one that I've read up to this point on the issue of creativity.

Suler suggests that psychoanalytic theory can be used for the purpose of devising a cognitive theory to explain creativity. Rather than view primary process thinking in terms of a regression from the more developmentally advanced secondary process thinking, he proposes that primary and secondary process be studied as relatively independent cognitive functions that interact in various ways.

There are perhaps two ways in which primary process thinking can influence creativity. The creative process can involve a temporary but direct access to primary process thinking for the purpose of using that ideation in generating creative insights. Creativity may also be mediated by cognitive activities that are derived from the permanent incorporation of primary process styles into stable secondary process operations.

In the article, Suler distinguishes between artistic and scientific creativity and discusses the differences and similarities between them. He cites empirical evidence supporting the hypothesis that creative individuals have greater access to primary process thinking. The article includes an extensive bibliography.

H.A. Witkin, R.B. Dyk, H.F. Faterson, D.R. Goodenough, S.A. Karp
Psychological Differentiation: Studies of Development
John Wiley & Sons, Inc.: New York, 1962.

After reading the article by Wormack, which found a correlation between field dependence/independence and physics achievement, I decided to look further into the theory behind this work.

An individual's degree of differentiation can be determined by a series of perceptual tests, such as the rod and frame test and the embedded figures test. These tests determine the individual's ability to differentiate parts from the whole. Highly differentiated individuals are called field independent, while less differentiated individuals are field dependent, tending to view the world in more global terms.

Witkin, et.al., suggest that young children tend to perceive in a relatively field-dependent fashion with a tendency toward field-independence increasing with age. Increasing differentiation could be seen as a basis of development. Yet, they also admit that the more field-dependent children become the more field-dependent adults, while the more field-independent children remain at the more field-independent end of the spectrum as they mature. In addition, the degree of differentiation of an individual tends to remain fairly stable across adulthood, with no apparent effect of trauma, training, or drugs.

In this book many studies are described which attempt to relate degree of differentiation to many different aspects of the personality. The authors found a significant correlation between ability to differentiate and problem-solving ability. Field-independent individuals were more successful at solving insight problems and at overcoming the Einstellung set.

The authors tend to describe a phenomena rather than suggest ways in which children can be taught to be more field-independent in their educational endeavors. They spend a great deal of effort trying to find out what the mothers of field-independent and field-dependent children are like. They offer no suggestions as to how these very different children can best be dealt with in the classroom.

Leslie Wormack
Restructuring ability and patterns of physics achievement.
Perceptual and Motor Skills, 1979, 48, 451-458.

This article was hard for me to follow because I did not quite understand the statistics used or the tests involved. The author used various tests of field-independence/-dependence, or restructuring ability, and found that they predict a person's ability to solve problems involving analysis and synthesis, including achievement in physics. For example, field-independent persons with high ability in spatial visualization attained a significantly higher average level of physics achievement than field-independent persons with low ability in spatial visualization.

D.I. McCallum, I. MacFarfane Smith, and J. Eliot
Further investigation of components of mathematical ability
Psychological Reports, 1979, 44, 1127-1133.

Found an association between spatial ability, which shows little relationship to comprehension of language, and understanding of mathematics.

Rodney L. Doran and Mouya K. Ngoi
Validation of a model for concept attainment levels with selected elementary school science concepts
Child Study Journal, 1976, 6, 21-32.

The authors suggested and tested three levels involved in attainment of scientific concepts: (1) recall of conceptual explanations for natural phenomena, (2) application of conceptual explanations to unfamiliar phenomena, and, (3) extension of conceptual explanations to a general class of phenomena. Testing sixth-graders at each level of concept attainment, they found that while the mean score decreased from level one to level two, it did not decrease from level two to level three, thus bringing into question the validity of the model.

Norman R.F. Maier and Ronald J. Burke
Test of the concept of "Availability of functions" in problem solving

"Function" refers to the specific uses particular objects serve in the solution of a problem. The purpose of this study was to further test the notion that problem solving is made easier when the necessary function is available in the subject's repertiore. After attempting to solve the Hatrack Problem, subjects were given the "availability of function" test. They were shown the correct structure and asked to list as many uses for it as possible. Results indicated that degree of success in solving the problem had no relationship to the subjects' availability of functions.

Norman R.F. Maier and Junie C. Janzen
Functional values as aids and distractors in problem solving

The two-string problem was used in this study, which allowed for a variety of solutions. The purpose was to test the effect of functional values of objects on problem solving. Results indicated that objects with more obvious functions served to distract the subject from finding innovative solutions, while those objects with functions totally unrelated to the problem situation tended to induce more innovative solutions.

Part IV: Theoretical considerations

In this section Maier attempts to isolate the variables relating to successful problem-solving behavior. These include an ability to not be misled by "obvious" solutions, a high frustration threshold, and constructive motivation. He cites variability as one of the most important factors involved in successful problem solving. An individual must be willing to try a variety of approaches, recognizing when it is no longer fruitful to stay with a particular solution. Another necessary ability is that of being able to unlearn or break up learned patterns to make elements of these patterns available to the solution of the ~~the~~ problem. Maier further postulates 8 selector-integrator mechanisms involved in the problem-solving process. These include the attitudes and needs of the problem solver as well as characteristics of the situation, such as the locus, intensity, and form of the stimulus.

Woodworth, Robert S. and Schlosberg, Harold
Experimental Psychology
New York: Holt, Rinehart, and Winston, 1954

Problem Solving: Thinking

Define a problem as existing when there is a goal, but no clear or well-learned route to the goal. Thinking occurs when explorations go beyond immediate situation and utilize memories and previously formed concepts.

Controversy as to what kind of imagery occurs during problem solving--verbal, visual, or kinesthetic.

Luchins' work on the inhibitory effect of a set during problem solving is cited. Woodworth describes Ruger's attempts to overcome this problem, by either asking subjects to formulate their assumptions, or by laying aside the problem for awhile. "Laying aside a problem is a means of getting rid of a false set or 'direction' and so giving the true direction a chance to emerge." The incubation period allows time for an erroneous set to die out, leaving the thinker free to take another approach to the problem.

Jerome S. Bruner
Beyond the Information Given: Studies in the Psychology of Knowing
New York: W.W. Norton & Co., 1973.

J. Bruner
The conditions of creativity

Bruner defines a creative act as one which produces effective surprise. Effective surprise has the quality of obviousness about it when it occurs, producing a shock of recognition after which there is no longer astonishment. All forms of effective surprise grow out of combinatorial activity--placing things in new perspective; effective surprise takes one beyond common ways of experiencing the world. Combinatorial acts that produce effective surprise almost always succeed through the exercise of technique.

J. Bruner
The course of cognitive growth

Bruner sees maturation as the orchestration, or combination, of simple acts into more of an integrated sequence. The integrated plans reflect the routines and subroutines that the child learns in a social environment. In order to combine these simple acts, the child must be able to represent them in some way.

Again, Bruner lists the three modes of representation--enactive, iconic, and symbolic. One is dependent on the former for its development; yet, they remain intact throughout life.

The rest of the chapter deals with conservation experiments. Bruner found that by using screens children performed significantly better on the tasks because, theoretically, they were forced to use a symbolic mode of representation rather than an iconic mode.

J. Bruner
The growth of representational process in childhood

Bruner lists three criteria that must be met in any theory of intellectual growth. It must characterize the operations of the mind in some formal and precise fashion. It must take account of natural ways of thought. It must take into account the nature of the individual's culture.

Bruner attempts to chart the development of representation. He cites three kinds: enactive, iconic, and symbolic. Enactive representation involves the actions an event requires, knowing something through doing it. Iconic representation involves forming some kind of picture, and symbolic representation requires that one translate an event into arbitrary symbols, or words.

Bruner discovered three stages in the development of representation, by means of an experiment that required children to discover which of two patterns existed in a light array. The 3-year-old child simply searches the board for bulbs that will light up. Bruner postulates that this indicates the strong interdependence of action and percept--the child attempts by his actions to acquire the solution. The second stage is exemplified by the 5-year-old child. The child brings a perceptual representation to the task, and tries out each image one at a time, sometimes accepting a solution on insufficient evidence. The older child of 8 years can successfully find the solution by means of an information-selection strategy which requires the use of symbolic representation.

Bruner cites another experiment utilizing a conservation task. He found that 6- and 7-year olds failed when they were required to base their judgments on iconic representation alone. They succeeded when they utilized both enactive representation, manipulating the clay, and symbolic representation, labelling.

Patricia Markds Breenfield and Jerome S. Bruner
Culture and cognitive growth

One interesting point from the article-- Piaget has described a form of child's thought as animistic--attributing inner psychological phenomena to inanimate features of the external environment. Bruner proposes that this mode of thought is not a developmental universal. Rather, in traditional, collectively oriented societies children see the world as realistic rather than animistic. According to Bruner, animism does not develop where there is no support given for individualistic orientation.

The remainder of the article deals with how cultural differences affect cognitive growth, with a rather extensive review of the literature dealing with the effect of language on cognition.

J. Bruner
The act of discovery

Bruner describes discovery as a matter of rearranging or transforming evidence in such a way that one is enabled to go beyond the evidence so reassembled to additional new insights. Often, the act of discovery is not dependent on the acquisition of new information.

There exists a dimension of cognitive activity that can be described as episodic empiricism vs. cumulative constructionism. In playing the game of Twenty Questions, trying to determine why a car went off the road and hit a tree, some children will ask questions that are so specific that a negative answer is unlikely to give them any new information, while other children ask more general, constraining questions, that serve to narrow down the field of possibilities. Bruner suggests that emphasis upon discovery in learning will help the child to become a constructionist, to organize information in order to discover regularity and relatedness and to avoid an information drift that fails to keep an account of the uses to which information may be put.

J. Bruner
Readiness for learning

Bruner hypothesizes that any subject can be taught effectively in some intellectually honest form to any child at any stage of development. One must represent the structure of the subject in terms of the child's way of doing things. Later on, the child's understanding of a concept will be more powerful and precise by virtue of this early learning. This article gives a description of the course of intellectual development and some suggestions about teaching at different stages in it.

Bruner describes three processes involved in the act of learning. First is the acquisition of new information; second is transformation--manipulating knowledge to make it fit new tasks; and third is evaluation, checking whether the way in which the material was manipulated was adequate to the task.

Jerome S. Bruner and Helen J. Kenney
Representation and mathematics learning

I think that this article and the preceding one are important from the point of view of barriers to learning. What Bruner and Kenney do in this article is suggest ways in which one can teach a child various mathematical concepts at earlier ages than one might think possible, by teaching them the material at their level. What impressed me is the thought that by teaching them certain principles through the manipulation of objects rather than ideas, when they reach the abstract stages it will probably be much easier for them to grasp the material. Possibly one reason why some individuals have so much difficulty learning mathematics is that they are never given the opportunity to learn the principles in the enactive or iconic modes before moving on to the symbolic mode.

Norman R.F. Maier
Reasoning in humans: I. On direction
Journal of Comparative Psychology, 1930, 10, 115-143.

Gave subjects a problem-solving task. Some subjects were given the necessary parts, or experiences, for solving the task while others were given "direction." Another group was given both experience and "direction" while the control group received neither. It was found that only the group receiving both experience and "direction" could successfully solve the problem, indicating that experience alone is not sufficient. A problem is that Maier is not real clear as to what is meant by "direction." In this case he indicated that the problem would be easy if one could hang a string from the ceiling. The solution involved wedging a wooden structure between floor and ceiling in order to construct two pendulums.

Maier also found that "direction" served as a more effective clue if given at the beginning rather than after the subject had made several attempts at solving the problem.

Norman R.F. Maier
Reasoning in children
Journal of Comparative Psychology, 1936, 21, 357-366.

Children were allowed to explore a maze in the shape of a swastika with a booth at each end. Then they were taken to one booth via an outside entrance where there was a desirable toy. The point of the experiment was to determine the child's ability to combine two experiences, the exploration experience and experience of the goal, to find his way from one booth to the booth containing the toy. The results suggest that children lack this ability to reorganize past experiences until about 70 months of age.

Lev Semenovich Vygotsky
Thought and Language
Cambridge, Mass.: The M.I.T. Press, 1962.

"Development of Scientific Concepts in Childhood"

Vygotsky makes the distinction between conscious and nonconscious concepts--often a child may correctly utilize a concept without being aware of what that concept is. According to Vygotsky, concepts may first be acquired at either level. Everyday concepts are usually acquired spontaneously, while scientific concepts are acquired consciously. He says:

> Scientific concepts, with their hierarchical system of interrelationships, seem to be the medium within which awareness and mastery first develop, to be transferred later to other concepts and other areas of thought. Reflective consciousness comes to the child through the portals of scientific concepts. (p. 92)

Vygotsky examined the temporal relation between the processes of instruction and development of the corresponding psychological functions. He found that instruction does not necessarily correspond to development. Instruction follows a rigid timetable involving a series of steps; often, each step will add nothing to a child's developmental progress. However, at some point the child will grasp a general principle and his developmental curve will rise markedly. One step in the instructional process may serve as a decisive developmental turning point.

Vygotsky is postulating two separate paths of development for everyday and scientific concepts. He suggests that the difference is between development starting at the bottom, with everyday usage, and moving up, to an understanding of the concept at an abstract level, as opposed to starting at the top and moving downward, for scientific concepts. He describes a study where children are required to finish sentences containing the words "because" and "although." Even though the children use these words correctly in their spontaneous speech, they have problems when required to use them consciously to describe everyday events, but not when describing social science concepts. Vygotsky cites this as evidence that as long as the curriculum supplies the necessary material the development of scientific concepts runs ahead of the development of spontaneous concepts. I think this is an interesting idea, but see one problem with the conclusion. The social science concepts could be so ingrained in the child that the sentences are completed from memory rather than from an understanding of the material. (The example Vygotsky presents is, "Planned economy is possible in the USSR because there is no private property...). However, in spite of this criticism I think that Vygotsky's ideas certainly should be taken seriously.

·Calvin W. Taylor (ed.)
Creativity: Progress and Potential
New York: McGraw-Hill, Inc., 1964

This book is not real helpful, although there is an extensive bibliography which, if this approach is pursued further, might prove of some value. What I am interested in is how creativity is involved in the learning of science. Taylor's concern is with identifying and nurturing creativity. He is apparently worried about our survival in international competition and sees the encouragement of creativity as the answer to our problems.

Taylor cites two working definitions of creativity, those proposed by Ghiselin and Stein. Ghiselin suggests that the measure of a creative product be the extent to which it restructures our universe of understanding. Stein defines a process as creative when it results in a novel work that is accepted as tenable or useful or satisfying by a group at some point in time.

In an article by Taylor and Holland, "Predictors of Creative Performance," the authors cite some general characteristics of the creative individual. Several, such as originality, redefinition, adaptive flexibility, and spontaneous flexibility, may be characteristics essential for the understanding of science. Another characteristic is the ability to sense problems. One individual once noted that part of Einstein's genius could be attributed to his inability to understand the obvious. I think the major problem with this chapter is that the authors never explain how they identified the creative person in the first place; I'm wondering if their definitions might not be somewhat circular.

E. Paul Torrance contributes a chapter entitled "Education and Creativity," in which he describes various stages in the developmental process of creativity. Two points in the chapter were interesting. Torrance lists several of the reasons why teachers may be reluctant to encourage creativity. Children may propose unexpected solutions which could disconcert the teacher; allowing the child to be creative may take more time; children may ask questions that the teacher cannot answer. Torrance also suggests that one way to encourage creativity is to challenge the child with problems that may be just a little beyond his grasp.

J.H. McPherson contributes a chapter, entitled, "Environment and Training for Creativity," most of which has little relevance. What is interesting, however, is his description of synectics theory. "Synectics" means the joining together of different and apparently irrelevant elements, and synectics theory attempts to describe the creative process. W.J.J. Gordon postulates four psychological states involved. "Detachment-involvement" requires that the individual remove a problem from its usual context and then become involved with it. "Deferment" involves resistance to the first solution that comes to mind. "Speculation" involves letting the mind run free and "autonomy of the object" is what happens when ideas crystallize and develop a life of their own. In order to achieve these psychological states it is important to make the familiar strange, and one can do this by means of personal analogy, direct analogy, symbolic analogy, and fantasy.

Karran P. Raghubir
The Laboratory-investigative approach to science instruction
Journal of Research in Science Teaching, 1979, 16, 13-17

Comparison between two laboratory teaching techniques: the Laboratory-Investigative approach and the Lecture-Laboratory Approach. Found that the former technique resulted in significantly better performance for both cognitive factors and associated attitudes (curiosity, openness, responsibility, satisfaction). Subjects were 12th-grade biology students.

Dorothy Gabel and Peter Rubba
Attitude changes of elementary teachers according to the curriculum studied during workshop participation and their role as model science teachers
Journal of Research in Science Teaching, 1979, 16, 19-24

Found that one method of changing elementary teacher's attitudes toward science teaching is through participation in workshops on new science curricula. However, the change is not stable over time, and there was no difference in later attitudes between teachers who served as model science teachers for preservice teachers and those who did not.

Charles Burrows and James R. Okey
The effects of a mastery learning strategy on achievement
Journal of Research in Science Teaching, 1979, 16, 33-37

Found that the mastery learning strategy, which provides individualized, self-paced instruction program with clearly stated objectives, test items, diagnostic tests, and remedial work, significantly improved student achievement. Only slight improvement occurred when either objectives or test items were provided.

Joseph Nussbaum
The effect of the SCIS's "Relativity" unit on the child's conception of space
Journal of Research in Science Teaching, 1979, 16, 45-51

Found significant improvement of child's understanding of space when taught "Relativity" unit. Larger percentage of transitions to higher cognitive stages found among pupils who were,at the beginning, at a more advanced level in conception of space ("needed" instruction less).

Richard J. Bady
Student's understanding of the logic of hypothesis testing
Journal of Research in Science Teaching, 1979, 16, 61-65

Confirmed evidence that high school students donot understand the logic behind hypothesis testing. Students tend to look for evidence confirming the hypothesis rather than evidence to disconfirm it. They also interpret implications as biconditionals (all p's are q's = all q's are p's).

Mohammed A. Kishta
Cognitive levels and linguistic abilities of elementary school children
Journal of Research in Science Teaching, 1979, 16, 67-71

Comparison of linguistic performance of students in three grades exhibiting different levels of performance on five selected Piagetian tasks. Some support for notion that language ability is indicative of underlying cognitive structures.

Ellen W. Fuller, David H. May, David P. Butts
The science achievement of third graders using visual, symbolic, and manipulative instructional treatments
Journal of Research in Science Teaching, 1979, 16, 129-136.

Results failed to provide evidence that students' gender or method of instruction influences students' ability to acquire or retain science concept of life cycles. Reading performance of students does appear to be a significant variable.

Leslie Wormack
Restructuring ability and patterns of science achievement
Journal of Research in Science Teaching, 1979, 16, 145-151.

Subjects who are proficient in perceptual restructuring tasks attain significantly higher mean achievement scores in science, in knowledge of science, and in ability to apply what has been learned in unfamiliar contexts, than students with low restructuring ability. Also tend to attain higher than average scores in ability to interpret science readings and analyze problems with science content.

Walter E. Lowell
A study of hierarchical classification in concrete and abstract thought
Journal of Research in Science Teaching, 1979, 16, 155-262.

Used Piaget-type tasks to assess cognitive abilities of junior- and senior-high school students. Found that those individuals dependent upon concrete referents for solving problems experienced difficulty on a hierarchical classification task, while those students who were not dependent on concrete referents experienced little difficulty. Interesting how few students in this study, 8 out of 60 high school students, were capable of solving problems requiring abstract reasoning ability.

Donald A. Vannan
Adapted suggestology and student achievement
Journal of Research in Science Teaching, 1979, 16, 263-267.

Attempted to adapt the "suggestology" method of Dr. Lozanov, from Bulgaria, to the teaching of elementary science to college juniors and seniors. Found that using methods to relax the students (comfortable chairs, instructing the students in autogenics--a method of relaxing the body--before the lecture was presented, presenting the lecture in various levels of intonation, then again with background music) resulted in a significant increase in the percentage of A grades achieved by the students.

Karran P. Raghubir
The effects of prior knowledge of learning outcomes on student achievement and retention in science instruction
Journal of Research in Science Teaching, 1979, 16, 301-304.

Found that presenting high school biology students with clearly stated learning objectives prior to teaching the science concepts resulted in improved performance on achievement tests, better retention of material learned, and greater understanding of cognitive behaviors higher than the knowledge level.

Abraham Blum
The remedial effect of a biological learning game
Journal of Research in Science Teaching, 1979, 16, 333-338.

Found that students in a sophomore course in phytopathology who played a learning game enhanced their understanding of the life cycle of Crown Rust.

Norman R.F. Maier
Problem Solving and Creativity
Belmont, California: Brooks/Cole Publishing Co., 1970.

PartI: The Search for a Creativity Mechanism

The studies in this section used the general format of giving the subjects lists of paired words (or, in the last study, simply a list of words) to memorize and then having them write stories using as many of the words as possible. Maier, et. al., theorized that creative persons would tend to either fragment or reorganize word pairs. The results indicated no correlations between measures of creativity and ways in which the words were utilized in the stories. However, individual differences were found, with some subjects staying with the pairs as they were learned, some predominently using single words (fragmenting) and still others forming new combinations of the words to suit the needs of the story. In a follow-up study Maier, et. al., found that introducing a decreased motivational factor for using word-pairs as they were memorized resulted in an increase in fragmentation, but not in reorganization. They concluded that individual differences in the reorganization of learned information are due primarily to differences in ability rather than preference.

Part II: Factors Influencing Success in Solving Problems

Melba A. Colgrove
Stimulating Creative Problem Solving: Innovative Set

Colgrove had two groups perform a similar problem situation. Both groups received the same instructions, with the experimental group being told that they were chosen because of their abilities to be original and to solve difficult problems. The experimental group generated significantly more innovative solutions than the control group.

Norman R.F. Maier and Junie C. Janzen
Are good problem-solvers also creative?

Compared subjects' performance on problems requiring one, correct, solution and on problems requiring any number of creative solutions. Found that those subjects who suggested a more creative, integrative, solution on the one task performed significantly better on the objective problems than those subjects who were not rated as creative.

L. Richard Hoffman, Ronald J. Burke, and Norman R.F. Maier
Does training with differential reinforcement on similar problems help in solving a new problem?

Subjects were divided into three groups, prior to solving the Hatrack Problem (building a hatrack from two boards and a clamp; the solution consisted of wedging the boards between ceiling and floor, clamping them together, and using the clamp as a hanger). The two experimental groups were given experience building a hatrack where they were allowed to use corners, pipes, etc., to aid in construction; the control group was given no practice prior to the problem-solving situation. One experimental group was given positive reinforcement during the practice session while the other was given negative reinforcement. The control group was significantly more successful at solving the problem than either of the experimental groups, and the two prior experience groups did not differ significantly from one another in their ability to solve the problem.

Ronald J. Burke, Norman R.F. Maier, and L. Richard Hoffman
Functions of hints in individual problem-solving

Gave subjects one of two different hints for solving the Hatrack Problem, either before they attempted to solve the problem or after 30 minutes. The hints were more effective when given at the outset of the problem-solving session. In general, they were found to have several effects: 1. Stop ongoing direction; 2. Serve as stimulant for correct solution; 3. Become absorbed or modified by ongoing direction; 4. Set up false directions; 5. Remain in background as point of orientation.

Ronald J. Burke and Norman R.F. Maier
Attempts to predict success on an insight problem

Used 18 tests to attempt to predict success on the Hatrack Problem; found no significant correlations. Discarded the possibility that solution was achieved by blind trial and error, since subjects appeared to pursue certain directions, and increased time did not increase the number of correct solutions produced.

Part III: Equivalent Stimuli and Functional Values in Problem Solving

Norman R.F. Maier
Reasoning in humans: III. The mechanisms of equivalent stimuli and of reasoning

Subjects were divided into three groups prior to solving Hatrack Problem: one group was shown a relevant structure which was subsequently left in place; one group was shown a relevant structure which was subsequently removed; the control group had no prior experience. With 25 subjects in each group, the numbers of subjects achieving the correct solution were 18, 12, and 5, respectively.

Norman R.F. Maier
Reasoning and learning
Psychological Review, 1931, 38, 332-346

Defines reasoning as the process of integrating two or more isolated experiences. Integration depends upon end or goal. Insight is defined as experience organism has when two or more isolated experiences come together; sudden experiencing of new relations.

Discusses the notion of a field of strain. Apparently subjects who are allowed to complete a task, either with or without interruption will not be able to recall the task as well as those subjects who are not allowed to complete the task. In reasoning the field of strain is set up by the desire to solve the problem and knowledge of the end, the attainment of which offers certain difficulties.

P.C. Wason and P.N. Johnson-Laird
Thinking and Reasoning: Selected Readings
Baltimore: Penguin Books, 1968.

H.G. Birch and H.S. Rabinowitz
The negative effect of previous experience on productive thinking

Used the two-string problem, allowing for only a single solution, that of a pendulum utilizing either a switch or relay as weights. Found that when subjects were trained, previous to the problem-solving situation, in the use of one of the items as an electrical device, they used the other item to solve the string problem. Perceiving an object as an electrical object made it extremely difficult for the subject to perceive it in its more general characteristic of mass.

R.E. Adamson
Functional fixedness as related to problem solving: A repetition of three experiments

Gave 2 groups of subjects the same problems to solve; for the experimental group the objects necessary for solution were presented in such a way to suggest a function different from that necessary for reaching a solution. The experiemntal group performed significantly more poorly than the control group on all three tasks.

P. Saugstad and K. Raaheim
Problem-solving, past experience and availability of functions

Proved that by insuring that subjects were aware of the functions of an object necessary for solving a problem, they would then be able to succeed at finding the solution. This particular experiment seems obvious, set up to get positive results. Saugstad and Raaheim made sure that subjects were aware that a bent nail could be used as a hook and that a rolled-up newspaper could be used as a funnel, then presented subjects with a situation where they needed to know those functions. Needless to say, the experimental group was significantly more successful than the control group.

A.S. Luching and E.H. Luchins
New experimental attempts at preventing mechanization in problem-solving

This was an interesting article, especially as it related directly to something that was brought up in the seminar. Luchins and Luchins have done a series of experiments that deal with the problem of habits mastering the individual, rather than vice versa. When an individual finds a certain technique to be useful in solving several problems, that technique gets carried over to subsequent problems, even though it may not be the most efficient method. Luchins and Luchins describe several attempts at preventing or reducing the effects of an Einstellung, or special kind of mental set.

Subjects are presented with 3 containers with varying volumes, from which they must obtain a specific amount of liquid. The first 5 test problems are solvable by the formula b-a-2c; the next two can be solved by the same formula, but also by a-c and a+c, respectively; the eighth can only be solved by a-c and the last two are similar to the sixth and seventh. Several variables were introduced. In one experiment subjects were told that they had only a limited amount of liquid to work with (the Einstellung method required more fluid). Even though subjects were specifically told to keep track of how much liquid they had, many did not, or else ignored the amount and went on to solve the last problems even though they had nothing left to work with. (These were pencil and paper tasks). Giving subjects an added incentive to abandon the Einstellung method did not appear to have a significant effect on their problem-solfing behavior. Adding a fourth, superfluous, jar in an attempt to get subjects to think about the problem more, resulted in an increase in direct solutions but also in failures and inefficient solutions. A third variation was to concretize the task, having subjects actually pour water from one jar to the next while solving the problem. This did not reduce the Einstellung effect.

F.C. Bartlett
Adventurous thinking

Bartlett gave subjects the problem of finding a route through a system of maps, where not all maps were presented at once. He found that as an individual proceeded in a particular direction, he was less likely to backtrack, the further along he got. He also found that at no point does the thinker show a strong bias toward shortcuts, numerically few risks, or either/or situations.

A.D. DeGroot
Thought and choice in chess

DeGroot makes several interesting points about the chess master's ability to solve chess problems. The master, primarily through experience and, as a result of this experience he has a schooled and highly specific way of perceiving and a system of reproductively available methods in memory. XX

Some characteristics about problem-solving in chess:

1. it is non-verbal
2. it is thinking in terms of spatial relationships and possibilities for movement
3. thinker must be able to foresee possibilities for action and foresee results

Several similarities exist between problem-solving in chess and the process of empirical research:

1. progressive deepening of investigation--ideas recur more than once; solution proposals tested with increasing thoroughness and compared with one another.
2. decisiveness of quantitative moment--no *a priori*, XXXX objectively fixed limit to amount or degree of improvement; yet, goal remains throughout.
3. decisions based on necessarily incomplete evidnece.
4. relativistic attitude required--*a priori* nothing is accepted as true or taken for granted.
5. complexity of hierarchical system of problems and subproblems that individual must remain aware of.

P.C. Wason
"On the failure to eliminate hypotheses . . . "--a second look

Subjects are presented with a series, such as 2, 4, 6, and told to find the rule. The correct response is simply a mathematical series in ascending order. More often than not, subjects test their hypotheses by looking for confirming, rather than disconfirming evidence and suggest rules that are too specific. For example, in testing the rule of even numbers with increments of two, a subject will state that 10, 12, 14 is an example of the rule and will actually not learn anything new.

K.J.W. Craik
Hypothesis on the nature of thought

Suggests 3 steps in the reasoning process:

1. "Translation" of external process into words, numbers, or other symbols.
2. Arrival at other symbols by a process of "reasoning," deduction, inference, etc.
3. "Retranslation" of these symbols into external processes or at least recognition of the correspondence between these symbols and external events.

U. Neisser
The multiplicity of thought

Suggests two thinking processes: the multiple process and the sequential process. Multiple process thinking occurs when an individual's awareness is divided between coexisting trains of thought, not always on a conscious level. This process encompasses intuitive, creative, productive, and autistic thought. Sequential thinking is a step-by-step process, utilized in reasoning.

John D. Roslansky
Creativity
Amsterdam: North-Holland Publishing Co., 1970

Jacob Bronowski
The creative process

Some points from this chapter:

Every induction is a speculation and it guesses at a unity which the facts present but donot strictly imply.

A man becomes creative when he finds a new unity in the variety of nature. The creative mind is a mind that looks for unexpected likenesses.

The difference between the arts and the sciences lies not in the process af creation, but in the nature of the match between the created work and your own act of re-creation in appreciating it.

Donald W. MacKinnon
Creativity: a multi-faceted phenomenon

This might have been a good chapter; too many pages were missing from the book for me to know for sure.

MacKinnon breaks down creativity as follows:

1. the creative process
2. the creative product
3. the creative person
4. the creative situation

The creative process has several stages:

1. a period of preparation
2. a period of concentrated effort
3. withdrawal
4. a moment of insight
5. a period of verification, evaluation, elaboration, and application of the insight.

One interesting point that MacKinnon makes is that the roots of creativity lie in the awareness that something is wrong, lacking, or mysterious.

Jerone S. Bruner
Beyond the Information Given: Studies in the Psychology of Knowing
New York: W.W. Norton & Co., 1973.

On perceptual readiness

Some of the issues discussed in this chapter might relate indirectly to the problem.

Bruner suggests that perception is in some respects similar to the cognitive task of categorizing: one moves inferentially from [illegible] cues to categorial identity. In addition, the perception of an object allows one to go beyond the immediately perceivable properties to a prediction of other properties not yet tested.

Bruner defines perceptual readiness as the relative accessibility of categories to afferent stimulus inputs. Two general determinants of category accessibility: 1) likelihood of occurrence of events learned by the person in the course of dealing with the world and, 2) requirements of search dictated by need states of the individual.

Perceptual readiness that matches the probability of events in one's world can be brought about in two ways: 1) by relearning of categories and expectancies or 2) by constant close inspection of events and objects.

This article actually covers a good many of the issues involved in perception and perceptual categorizing. One other point he makes regards the ways in which failure of perceptual readiness may come about: 1) through~~t~~ failure to learn appropriate categories and, 2) through a process of interference whereby more accessible categories with wide acceptance limits serve to mask or prevent the use of less accessible categories.

Jerome S. Bruner, Michael A. Wallach, and Eugene H. Galanter
The identification of recurrent regularity

A study of possible factors influencing subjects' abilities to correctly identify a right-left pattern in lights. One interesting finding was that subjects who responded from the outset, rather than those who were told to observe, through the first three sequences of the pattern, and to pay attention to the stimulus, did more poorly on the task. Bruner, et.al., also point out that, in identifying environmental regularities, effective advice would be to, "Pay attention to the stimulus and disregard your past responses." Theories of reinforcement donot explain how a person cuts through the interfering properties of the environment when such exist and when identification is not immediate.

C. Rodney Killian
Cognitive development of college freshmen
Journal of Research in Science Teaching, 16, 347-350, 1979.

Found that, out of 106 college freshmen, 25% were reasoning at the cognitive level of formal operations, 60% were at a transitional level, and 15% at the concrete level.

Joseph R. Riley, II
The influence of hand-on science process training on preservice teachers' acquisition of process skills and attitude toward science and science teaching
Journal of Research in Science Teaching, 1979, 16, 373-384

Found that two treatments, training in process skills using a manipulative, "hands-on" approach and receiving vicarious training with the same method (no student manipulation of science materials), resulted in significant improvement in preservice teachers' competence in selected process skills.

Max Wertheimer
Productive Thinking
New York: Harper & Brothers Publishers, 1945.

Wertheimer discusses various approaches to problem solving, both from a theoretical and from a practical point of view. On the theoretical side he describes the pitfalls in using either a traditional logical analysis or an associationistic theory to explain how an individual comes to solve a difficult problem. From a practical point of view he criticizes rote instruction, the teaching of formulas without adequate explanation, as being entirely unsatisfactory in giving the student an understanding of mathematical and scientific principles.

The format Wertheimer uses is to describe the learning processes underlying various mathematical principles such as finding the area of a parallelogram, proving the equality of vertical angles, and finding the sum of the angles of a polygon. He also theorizes on the thought processes that Galileo went through when discovering the laws of inertia and he discusses the various steps that led Einstein to the discovery of the theory of relativity.

Wertheimer makes some interesting points about problem solving, which are, however, rather vague. He lists some essential features of genuine problem solving as: 1. not to be bound by habits, 2. not to repeat slavishly what was taught, and 3. not to look at the problem in a piecemeal fashion. In order to successfully deal with a problem one must view the problem as a whole and attempt to realize how the problem and the situation are related. For example, in proving the equality of vertical angles, one must look at the entire structure: . The proof will not be found by separating the two angles: . In viewing a problem as a whole one must center, or focus on the objective structure of the situation. Wertheimer discusses some of the problems that can occur when one takes a one-sided, often egocentric approach. One can become confused when a person describes social relationships in terms of himself rather than in terms of the truly central person. To summarize, Wertheimer states:

> Productive processes are often of this nature: in the desire to get at real understanding, requestioning and investigation start. A certain region in the field becomes crucial, is focused; but it does not become isolated. A

> new, a deeper structural view of the situation develops, involving changes in the functional meaning, the grouping, etc., of the items...
>
> Two directions are involved: getting a whole consistent picture, and seeing what the structure of the whole requires for the parts.

P.C. Wason and P.N. Johnson-Laird
Thinking and Reasoning: Selected Readings
Baltimore: Penguin Books, 1968.

N.R.F. Maier
Reasoning in humans. II. The solution of a problem and its appearance in consciousness.

Attempt to answer the following questions: 1) Does the solution develop from a nucleus or does it appear as a completed whole? 2) What is the conscious experience of an individual just before the solution is found? 3) Is the reasoner conscious of the different factors which aid in bringing about the solution?

Used the two-string problem which allowed for several solutions, although the one Maier was looking for was using pliers as a weight and changing one string into a pendulum. This would involve changes in organization and meaning of the vectors in the problem, as discussed by Wertheimer.

Maier focussed his attention on the group of subjects who were able to successfully solve the problem after hints were given: 1) The experimenter made the string sway back and forth; 2) The subject was told that the problem could be solved by using the pliers and nothing else. Two types of experiences were found among these subjects--the solution was either experienced as a whole or in two steps (after each hint). Maier theorizes that in the first case, both hints were important, but that the first hint was not consciously experienced. It was also found that subjects tended to repeat variations of previous solutions during the problem-solving process.

Maier concludes that the results of the subjects throw no light on the nature of reasoning.

K. Duncker
On problem-solving

Duncker describes the processes underlying finding the solution to two different types of problems: 1) a practical problem of how to destroy a tumor with X-rays without destroying the surrounding tissue and, 2) a mathematical problem of explaining why all six-place numbers, of the form 276,276, 112,112, are divisible by 13. Unfortunately, I was unable to completely follow the theoretical discussion. He talks about the process of solution-finding as going from the original setting of the problem to the functional value, or principle, of the solution, to the more concrete forms of the solution. He uses the analogy of the "family tree" to describe this process.

Duncker also describes how the use of hints can aid in the problem-solving process. He found that the more concrete the hints, the more they helped. Would this indicate that it is easier to find a solution to match the problem rather than following the "family tree" hierarchy?

Duncker also discusses the necessity of "restructuring." One must be able to see numbers of the form abc,abc as being equal to abc x 1001, and 1001 as being a multiple of 13, in order to solve the problem.

APPENDIX B

Abstract from the Proposal
to the National Science Foundation

THE RATIONALE

I. Subject of Intended Research

The research proposed here deals with a specific class of learning difficulties in elementary science and mathematics. We have called these *critical barrier phenomena*; unless surmounted, they inhibit further learning, whereas when surmounted successfully, they conspicuously advance it.

A. Critical Barrier Phenomena

In previous publications we began to define and illustrate the nature of these critical learning phenomena[1] (Appendix A). The illustrations are derived from our teaching experience in the elementary physical sciences and biology at several age levels and from many backgrounds: pre-school and elementary teachers, pre-college and college students. We have informally confirmed the widespread manifestation of these phenomena, sometimes in precise detail, in discussion with other science and mathematics teachers who work with children, older students, and teachers. There is, therefore, an initial justification for the claim that the phenomena have some universality among the scientifically non-literate population.

Though the existence of individual critical barriers appears to have been recognized by many thoughtful and experienced teachers, they appear not to have been considered *collectively* as providing important clues to the improvement of the teaching art, to curriculum-making, or to cognitive science.

Example 1. A typical example of a critical barrier is the difficulty experienced by many people in understanding the operation of a siphon or suction pump, the height of a column of water in a jar inverted in a pan of water when the air has been partially withdrawn, or the liquid barometer. There typically develops an explanatory impasse, connected on the one hand with the *absence* of any awareness of the weight of the atmosphere per se, or as a possible factor in such situations, but connected on the other with the *presence* of some intuitive notion of the kind expressed long since by the maxim that "nature abhors a vacuum." With many of our adult students the situation is much as though Gallileo's and Torricelli's investigations had never taken place. One adult said, "Air is just not the sort of thing you could put on the scales!" For such persons

"The weight of air" has no intuitive meaning. In Aristotle's classification of the elementary forms of matter, air has *levity* not *gravity*. In our discussion with reflective adult students we repeatedly hear that their difficulty is one of becoming aware of their own preconceptions; when in the present case, they become aware of the *Aristotleian* frame in their own thinking they are able to compare it to, and for the first time to understand, the *Torricellian* frame, which requires that the liquid in the jar be seen as one part of a two-part equal-arm balance. This is a not inconsiderable conceptual achievement, with the column of water conceived as balancing that of an atmospheric column of air.

Example 2. A second typical example of a critical barrier is that of mirror vision. A barrier appears in situations calling for the prediction and interpretation of mirror-vision phenomena, and more generally of a wide range of phenomena involving the understanding of simple virtual and real images. Thus, when most children and scientifically naive adults are asked to represent or make predictions about oblique mirror vision they represent the line-of-sight to a mirrored object (diagram p.1., Appendix A) as if to a *picture* of the object on the surface of the mirror. In a preliminary demographic survey this representation characterized a sizeable majority of university graduate students in non-physical science fields, as well as undergraduates, elementary school teachers, and children. The picture hypothesis, which we infer from examining many geometrical drawings and from informal conversations, has been confirmed in our teaching practice in connection with quite other phenomena: for example, the surprising inversion of real images formed by lenses or pinholes, which are typically seen also as independently existing *pictures*, dissociated from any intuition of the light-ray projection. The interpretation of real or virtual images as phenomena wholly constituted by optical projection, in contrast with paintings or photographs, appears to require a conceptual frame not accessible to most scientifically naive adults and children.

Example 3. The abstractions of length, area, and volume in their geometrical meaning and quantitative representation appear as critical barriers in many elementary contexts. In the exercise of building bigger squares from square tiles, bigger triangles from unit triangles, bigger cubes from unit cubes, many adults appear to be in serious difficulty both with the geometrical concepts and wiht their arithmetical translation. Tabulations representing area and volume as functions of linear size are surprising and wholly unanticipated. For irregular shapes area and volume are conceptually ill-defined: area is often confused with "distance around," perimeter. A stubborn intellectual difficulty then also arises with the concept of surface-volume ratio and with any possible relevance it might have to qualitative aspects of biological and physical phenomena.

We see and interpret the world in terms of our accumulated experience as modified by the explanations offered by parents, peers, and teachers. Common sense has many conceptual resources: unsupported objects fall, solid objects retain their shape, while water spreads to conform to the shape of its container, and so on. These ideas are easily built up from everyday experience. Yet other, equally elementary but scientifically pervasive concepts are not accessible to common sense; the concept of the weight of air which explains many common phenomena, does not easily arise from everyday experience, nor do the concepts of balance, or of heat and temperature.[2] Commonsense frames of thought carry students so far but fail them at critical points; when they meet scientific explanations at school or college, but perhaps in informal settings, conflicts arise because they cannot reconcile formal scientific explanations with the intuitions of everyday experience. It is here that critical barriers to learning appear to arise.

Verbal explanations do little to resolve these difficulties, which arise not from lack of understanding a few facts but from the failure to develop appropriate, widely applicable conceptual frames. These are likely to be acquired only through varied -- and guided -- experience of the phenomena in question. Such frames must be constructed by learners, they cannot simply be installed from the outside in minds unprepared.

B. Definition

A working definition is that critical barriers:

1. Are conceptual obstacles which confine and inhibit scientific understanding.

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[2] Attached as Appendix D

2. are "critical" and so differ from other conceptual difficulties in that they:
    a. involve preconceptions which the learner retrieves from past experiences that are incompatible with scientific understanding.
    b. are widespread among adults as well as children, among the academically able but scientifically naive as well as those less well educated.
    c. involve not simply difficulty in acquiring scientific facts but in assimilating conceptual frames for ordering and retrieving important facts.
    d. are not narrow in their application but when once surmounted provide keys to the comprehension of a wide range of phenomena. To surmount a critical barrier is not merely to overcome one obstacle but to open up stimulating new pathways to scientific understanding.

II. Components of Proposed Research

A. Experimental Teaching

For the three-year period of the proposed research we intend to pursue one central line of research activity and to weave several others around it. This central line of research will be experimental teaching in specific science-math areas which we select because we know or suspect that a large fraction of students in the group taught will exhibit the difficulties we wish to investigate. The groups will range from elementary school through university students and elementary school teachers.

1. Aims. The aim in this experimental teaching will be (a) to identify and record conceptual obstacles to understanding, both those we have already anticipated as critical barriers and also new ones freshly observed; (b) to record, describe, and analyze our own responses -- as experienced teachers of very elementary science and mathematics -- to the discovery of such difficulties, and (c) to record the outcome of the teaching in terms of students' resulting performances, and hence to provide guidance to teachers and curriculum developers. The record of these experimental teaching episodes will enable us to document barrier phenomena in some detail. Thus in the example of mirror-vision, when interpreting or predicting mirror-image phenomena a majority of scientifically naive individuals of all ages appear, as we said above, to retrieve from memory a frame which relates such phenomena to some conceptual frame of pictures, and this

interpretation appears to be confirmed by responses of the same individuals to the observation of images in other connections, for example in matters as the inversion of pinhole images. This plausible hypothesis needs to be tested and confirmed or modified by carefully recorded evidence.

2. Methods. The experimental teaching will have the format of short courese, or sequences, in longer courses, of 8-15 hours. The elementary nature of the critical barrier phenomena we wish to study allows us to teach essentially the same subject matter across different age groups, allowing for differences in style and levels of verbal communication.

3. Outcomes. From the results of such teaching it is our aim to create a taxonomy of critical barriers as typically encountered in these elementary teaching contexts. We also intend to produce a parallel taxonomy of teaching techniques which can be used to anticipate, identify and overcome such barriers. These taxonomies will be supported by case histories of individual groups and students, including interviews. The context provided by such case histories will make this taxonomy speak both to further cognitive research and to the needs of curriculum developers and, more importantly, of science teachers.

B. Development of Theory

1. Scope. The nature and prevalence of these critical conceptual barriers in elementary science education is a subject for wider study than is possible exclusively in experimental classrooms. In the following sections we shall enlarge upon the need to elaborate an historical and theoretical context for barrier phenomena. Many of the conceptual difficulties which appear to stand in the way of widespread scientific literacy today are closely related to difficulties which have been apparent in the history of science itself; what is now considered elementary is often of historically recent development, reduced to familiarity only after major intellectual struggles, and then -- as our preliminary evidence strongly indicates -- only within the special subculture of the scientifically well-educated. A careful investigation of critical conceptual struggles in the history of various scientific disciplines should provide specific parallels to our

empirically observed critical barrier phenomena, and will give important clues to their interpretation. (cf. Appendix B)

A second phase of our proposed theoretical research is to relate specific critical barrier phenomena to the present state of knowledge in the epistemology and psychology of cognitive development. The conceptual frames which learners do or do not retrieve in attempting to understand simple natural phenomena are evidently not learned in the same way specific items of factual information are learned; they are habitual resources of thought rather than items of knowledge. They are thus developmental in character: their development is often reminiscent in tempo and style of those imputed to children in Piagetian research. An important difference, however, is that our critical barrier phenomena have been found in many people of all ages.

We believe that from the examination of critical barrier phenomena, both in historical and developmental contexts, other related empirical investigations will follow. One, which we definitely plan for, is a demographic sampling to demonstrate, in a quantitative way, the degree of prevalence of specific barrier phenomena among different population groups. We have already found one such sampling possible (Appendix A) and propose to develop relatively inexpensive means for sampling responses in other subject matter areas from relatively large groups of adults and children.

2. Research Seminar. The central vehicle of our proposed research, beyond the scope of experimental teaching, is that of a multidisciplinary research seminar. This seminar and the record of its deliberations will provide an historical and theoretical context for all of our work, and will include invited contributions from its members and visitors relating to:

1. the history of relevant scientific developments
2. the epistemology of science
3. cognitive development in childhood, adolescence and adulthood
4. means of laboratory and demographic research in 2 and 3
5. the discussion and analysis of findings and case histories from experimental teaching

We are fortunate in having at the University of Colorado a number of well-informed specialists in education, the history and philosophy of science, in developmental psychology, and in research on adult cognitive processes.

This seminar will also provide a vehicle, on our own campus and elsewhere, for the dissemination of research interests and findings within wider academic circles. We expect it to be a seed-bed for the expansion of research and practical effort in elementary science and mathematics teaching for college and pre-college students.

The work of the seminar will be affiliated with the newly-created Center for Interdisciplinary Studies at the University of Colorado in the College of Arts and Sciences. We expect that members of this seminar -- including specifically two members of the project staff -- will be involved in designing mathematics and science courses for future elementary school teachers attending the university. These courses will have a special status in our research designs, focusing on the materials, techniques, and conceptual background for elementary school science teaching. We regard this as a major opportunity to investigate, and to demonstrate, ways of breaking the vicious circle by which those who have been poorly educated in science and mathematics become the teachers of our children in their first exposure to these fields of knowledge.

III. Theoretical Background

The aim of science has always been to extend and refine our experience and to reduce it to a coherent system. But in this process it has frequently been necessary to re-examine and reconstruct previously received ideas and habitual modes of thought. For this reason many scientific ideas and modes of understanding, even those which are now regarded as "elementary," have evolved in fact over at most a few centuries, sometimes only generations or decades. They represent creative conceptual innovations. not simply new facts. Those who have grown up in the culture of present-day science may find them "obvious," but even in simple form they would have been inaccessible to investigators of an earlier time. Thus, for example, the physical and biological importance of simple size-scale relations is first anticipated in Galileo, still only implicit in Newtonian mechanics, and not fully recognized until the twentieth century. In the past thirty or forty years, fresh historical and philosophical scholarship has added much to the understanding of this history, which proves to be a complex intellectual evolution affecting as much the frame work of natural knowledge as its factual content.[3] The result suggests not simply the increase of factual knowledge, but also an evolutionary or genetic epistemology in which new factual content and rational framework evolve in interaction with each other. Most of this scholarly attention has been devoted to developments within the scientific culture or subculture. Interactions with society at large, with the majority culture, have been investigated mainly in connection with the co-evolution of technology and its institutions or in reference to specific and dramatic conflicts of belief engendered by scientific progress (Galileo, Darwin).

There is another sort of interaction of great importance which is still inadequately understood or investigated. On the one hand, scientific leaders have always voiced a commitment to the spread of scientific understanding to the whole of society. On the other hand, the very development of science has made that progress increasingly problematic in spite of many efforts of formal and informal popular education. The resulting and pervasive state of affairs was poignantly described by John Dewey a half-century ago: "Science is too new to be naturalized in experience. It will be a long time before it so sinks into the subsoil of the mind as to become an integral part of corporate belief and attitude."[4]

We believe that the full nature of the commitment to widespread scientific enlightenment, through formal or informal means, has seldom been adequately conceived. Early instruction and popularization have been conceived as transmission and diffusion of knowledge but there has been little systematic effort to understand the sources of indifference or resistance to the popular acceptance or assimilation of scientific style and content. There is little recognition that naive or commonsense mentality is not a vacuum to be filled with new knowledge, but a plenum already highly structured for accomodation to everyday phenomena of nature and social life. (See Appendix C, passim, esp. p. 13). Nor is there sufficient recognition that science can be well learned only by a two-way process of accomodation between these.

pre-formed structures and those of science. To define this interaction better one must first attempt to understand these commonsense structures and credit them with an appropriate range of validity, not treating them as mere expressions of ignorance or error to be replaced -- as the old term *enlightenment* suggests -- without internal conflict.

We must take account, of course, of the ways in which science *has* been accepted in the majority culture, specifically with regard to technological evolution. In the passage quoted, Dewey suggested that until the scientific subculture has "sunk into the subsoil of the mind," "both method and conclusion will remain the possession of specialized experts, and will exercise influence only by way of external and more or less disintegrating impact upon beliefs, and by equally external practical application." We accept television and the airplane but we almost totally lack an understanding of their elementary scientific basis.

In the proposed research we are concerned with the development of human intellectual resources that can be distinguished (not separated) from the acquisitions of empirical knowledge; we thus value the work of Piaget and his associates while at the same time we find it somewhat aside from our main direction. We agree with its basically interactionist view[5] of intellectual development, and with its broader framework of stages; we also recognize a genuine consonance between our critical barriers and some of the Piagetian critical phenomena, for example, conservation of number, mass and volume, seriation, and other organizing schemes. On the other hand, we have approached these kinds of phenomena from the point of view of education, whereas the Piagetians have focused on developments which appear to be relatively invariant to differences within the cultural ambiance in which they occur, and qualitatively insensitive to specific teaching efforts. We agree with Piaget (and with Plato!) that such developments are relatively insensitive to short-term efforts to teach them. But so far as we know, Piaget has never brought research attention to the study of adult commonsense conceptualization (See Appendix C, p. 8) as it develops in children or is predominantly present in adult life. We find, for example, a widespread failure among children *and* adults to grasp certain elementary geometrical invariants of scale: for a homely example, few people of can think correctly about the amount of water which could be added to a gallon container of sand versus a gallon container of coarse pebbles. Most (including one techincal chapter on soils) say more water is added in the second case, "because the holes are bigger," while some say less, "because there are fewer holes."

If we were to use parallels for the evolution of science from biological evolution we would emphasize both the rapid growth of the sciences within special social niches, niches which would have evolved with them, and at the same time their divergence from the evolution of the majority culture. Here the parallel is speciation rather than adaptation. Piaget's scheme on the contrary is unilinear, envisioning a history of improved adaptations which are then somehow incorporated in the culture and relived in individual development. Our scheme includes branching into different lines within a shared social ambiance, lines that are divergent and not automatically interactive. If we were studying a time when the marsupials were dominant over the still-rare later mammals, we would have to recognize that both were adapted for survival -- in different niches -- and that both illustrated the maxim that ontogeny recapitulates a common phylogeny, though only up to the time of their divergence.

Piaget's thesis, that the study of child development gives clues essential for the study of human cultural development, is thus tantential, not central, to our work. We preach no dogma of recapitulation; we wish only to refine and support the description of present-day problems of science education by fruitful historical comparison.

Our historical emphasis, however, differs from that of J.B. Conant who justifies science teaching via historical studies in a sense by reliving the past, and who has been responsible for numerous excellent historical case studies intended for use in college level general science courses. There are many pathways of intellectual development in elementary science education, and history suggests only some of these; others prove more accessible to present-day learners. We can create more royal roads to modern geometrical knowledge than that which starts with Euclid, even (and especially) at elementary levels. Thus the historical study of Greek mathematics may provide us with clues as to conceptual difficulties associated with problems of size and scale, but present-day children or adults can be helped to overcome these difficulties along many paths other than those of history. Our resources are wider.

IV. The Present Research Position

The concerns outlined above have often only been peripheral in the relevant research literature. When central, they are limited to special contexts. The relevant literature falls into four main categories:

A. A scattered literature dealing globally with philosophical perspectives on the evolution of science and evolutionary epistemology of science. This literature in important in helping to define what are usually referred to as conceptual schemes or frameworks, and in providing alternative accounts of their role and origins: empiristic, nativistic, and interactionist.[7]

B. A large literature in the historiography of science, sometimes describing the interplay of theory and observation out of which specific elementary scientific concepts and modes of thought have emerged,[8] including some of those related to our category of critical barriers. Some of this literature is relevant to our specific questions about parallels between the conceptual struggles of historical discovery and those of contemporary science teaching.

C. A research literature concerned with observed difficulties in the assimilation of scientific and mathematical knowledge by children and adolescents. Much of this research is concerned with the apparent existence of conceptual barriers to understanding in relation to curriculum organization, age or developmental readiness, transfer of learning; but not, as we propose, upon the description and explanation of a wide range of specific confusions or misunderstandings which are assumed to be responsible for failures in instruction.[9] Thus Ausubel[10] speaks of preconceptions which appear to block the growth of scientific understanding as "amazingly tenacious and resistant to extinction." Such research literature confirms the legitimacy of our focus on critical barrier phenomena. Its main concern, however, has been to measure the divergence between students' performance and some norm of adequate scientific understanding. The precise nature of students' attempts to assimilate what is offered in instruction requires a more qualitative, ideographic description, and is seldom investigated.

D. There is a related literature, largely associated with the name and tradition of Piaget, which seeks to describe the assimilation of elementary scientific and mathematical subject matter in terms of match-mismatch between the framework of contemporary thinking implicit in that subject matter on the one hand, and the alternative frameworks which habitually guide or control the thinking of the learner on the other. Such research is aimed at describing the latter developmentally. Though this Piaget thesis is not in theory a nativistic one,[11] research related to it pays little attention to the learning-teaching context as a source of developmentally relevant but controllable variables. It is restricted primarily to child-adolescent subjects and views adulthood as merely an extension of childhood rather than as an independent subject for empirical research. Our own work by contrast, is concerned as much with adult thinking and performance as with childhood. Critical barriers are prevalent -- with variations -- in both children and adults in forms which suggest strongly that cognitive development is inadequately described as a unilineal sequence of well-ordered and more or less age-dependent stages. There appear to be conspicuous and sometimes great differences among adult thinking processes about even very elementary science. These differences are associated with educational opportunity, differences in interest, factual knowledge, and the like, but they are not adequately explained by these.

The critical barriers we propose to investigate are rather clearly developmental in character, but they appear to us to be poorly described by what Piaget calls decalages, the unevenness of development across differences of age and subject matter. They seem rather to imply, especially in the case of otherwise well-educated and thoughtful adults, some interference between alternative pathways of intellectual development. As we have said, evolution implies divergence (speciation) as well as adaptation.

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V. Significance of Proposed Research

The proposed research is in an area of central concern in science education: the comparative examination of conceptual difficulties in the understanding and fluent use of pervasively important elementary scientific ideas and the habits of thought which their full assimilation can support. In comparing the conceptual expedients or habits of children, adolescents, and scientifically naive adults across the same ranges of elementary subject matter we believe we are breaking fresh ground, though ground adjacent to areas which have been increasingly investigated in the last two or three decades. Our guiding hypothesis of divergent intellectual evolution, cultural and individual, can, if well supported, be an initial challenge to the dominant developmental scheme which conceives such development to move, though perhaps at different rates, along a unilineal path. We hope that our work will be sufficiently cogent in theory and clear in empirical findings to merit attention from developmental psychology and educational research. Indeed, we plan this research to provide one sort of bridge between those areas. Whereas the conditioning hypothesis of unilineal development does not require an interactionist view, that of development which diverges from early ages clearly does: human development is a function of native endowments and of life-experience and choice within a socially diverse array of opportunities, wide or narrow. Our research clearly has that philosophical presupposition, one which supports the belief that development can be materially affected by appropriate educational intervention. The Piagetian school has tended to assert scepticism about the developmental contribution of short-term teaching-learning situations. We believe, however, that the interventions they have considered have typically been of a conventional sort, not sampled from a range of possible styles, strategies, and durations. Short of full-scale longitudinal studies, we believe that our focus on critical barriers in science learning will yield significant developmental and educational knowledge.

Our proposals for demographic, historical, and classroom studies have come primarily out of personal experience and reflection: the hoped-for outcome, even if conforming to our expectation, will still stand in need of possible qualification, extension, and replication.

The above are formal research outcomes that we hope for. For the local teachers we shall enlist as participants in the research the outcomes are a potential contribution not so much to research literature as to the state of the art. The history of educational research and development leaves major doubts about its contribution to the improvement of education. Like all the arts, that of teaching appears to be transmitted and improved best through reflective participation. Thus our school-level associations are to us important in their own right.* In the same way, we hope our enlistment of university faculty members will lay the basis for a continuing interdisciplinary research and development group concerned both with pre-college and college teaching, including especially the education of future teachers.

VI. Procedure

A. Research Seminar and Consultative Committee

The organization of the project will center around a multi-disciplinary research seminar and committee. This group includes the proposed research staff but also includes university faculty members with professional status in physics, biology, mathematics, psychology, the history of science, the philosophy of science, and educational research. Faculty participants hold senior academic positions and have been chosen both because of their relevant professional knowledge and their special personal commitment to the improvement of formal and informal education.

Members of this seminar will be responsible for contributions to its discussions from their fields and for planning specific aspects of the research.

The central role of this seminar or committee is justified by three main considerations:

(a) Its interdisciplinary composition matches the needs of the research: by providing:

  i) subject matter knowledge in the physical and biological sciences and mathematics.

  ii) professional knowledge of the history of science, and access to its historiographic literature;

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*(See Appendix E)

iii) knowledge and research experience in developmental psychology, with special reference to the ways in which children's thinking is dependent not only on developmental processes, but also upon the kinds of ambiance and access provided, in relation to which their behavior is observed;

iv) knowledge and research experience in the study of adult mental processes as related to intuitive, quasi-rational and rational modes of thought.

v) provessional educational research and its recognized methodologies.

vi) personal skill and reflective craft knowledge related to learning and teaching at various age levels and subject matter levels.

(b) The seminar itself is potentially a nucleus of an ongoing research group in the new Center for Interdisciplinary Studies at the University of Colorado devoted to science and mathematics education.

(c) In this seminar we hope to provide a link to research and practical development elsewhere. We plan to invite visitors to discuss their own work. (We already have exchanged visits with Paulo Guidoni and Matilde Missoni, physicists at the Istituto di Fiscia, University of Rome. Their research in many ways parallels our own (See Appendix C). We would also hope to benefit from the work and thinking of Frank Oppenheimer, Director of the San Francisco Exploratorium. We believe our work will have direct implications for informal science education, of which he is a principal exponent. We shall be in contact with Dr. Jack Easley of the University of Illinois, and others at the University of Washington, M.I.T., and in the United Kingdom and Holland. We shall also keep in touch with the other NSF projects which have a bearing on our research.

B. Experimental Teaching

(a) Short Courses. These courses will have two specific purposes, beyond that of observing and identifying commonsense-scientific dissonances. The first is to test hypotheses about the precise nature of specific commonsense modes of thinking. The second is to test our general belief that these barrier phenomena, once brought to the surface and consciously examined, can provide a predictably fruitful challenge to learning -- but only if, in the future, teachers are prepared to identify them and have a relevant repertoire of suggested responses. An outcome of this will be a taxonomy of teaching strategies.

The design involves the presence of a staff observer and the preparation of detailed notes by both teacher and observer immediately after each session. These will be supplemented by audio and video tapes for later analysis. This recording has two aims: The short-term aim is research in the service of teaching, *not* of publication. Diagnostic observations, shared by teacher and observer, lead to revision or amplification of short-term plans; when carried out these give further observable results tending to confirm or deny the previous diagnosis. Further short-term planning then ensues, and the cycle is repeated many times. Such differential planning is, of course, a component of good teaching.

The long-term outcome of such research, one which *is* for publication, can demonstrate the cumulative effects of this cyclic process. The diagnostic judgments, made and recorded in teaching and planning, are predictive and testable by observations accruing *after* the record of them is made. This cumulative record can be that of a self-correcting sequence of judgments and plans which are individually subjective and fallible, but collectively significant in the end result. This *process* is transferable.

Though it is not "teacher-proof," such research is, we believe, teacher-proven nad needs public research support. Such research does not in principle define sufficient conditions of successful teaching; but it can demonstrate what is involved in a style of teaching which many successful teachers would recognize as related to their own.

(b) *Program*

Topics

i) The Ocean of air
ii) Liquids
iii) Heat and Temperature
iv) Light and Vision

These clusters of topics are rich in critical barriers which are important in elementary science. (See Appendix A,B)

v) Size and Scale
vi) Number and Form
vii) Others to be encountered

We have already evolved a wide variety of teaching materials from these clusters which we shall use in our experimental courses.

(c) Target Groups

i) Elementary teacher study groups. In these we shall invite teachers to be both science students and investigators in discussion of their own and their students learning and learning difficulties. (5-10 teachers: 2 hours/10 weeks/term).

ii) Secondary teacher study groups. Similar to i)

iii) Students in elementary and secondary schools. With members of the above two groups we will arrange follow-up teaching in their own classrooms with members of our staff as teachers or observer/recorders. (3 hours/week: 5-10 weeks/term).

iv) Undergraduate non-science majors. Year long intensive courses in general science are being designed in the Center for Interdisciplinary Studies. Four faculty members of this program, members of our research seminar, will be involved as teachers and observer/recorders.

v) Elementary education majors. A special section of the above course is planned for elementary education majors. Our staff will play a special part in teaching and observing this group both in laboratory work appropriate to elementary school science and in the discussion of critical barriers and teaching strategies related to them.

(d) Teachers and Recorders

Undergraduate Courses	Director and Co-Director
Teacher's Courses	Faculty Members of Research Seminar
Schools	Project Staff, selected teachers

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(e) Schedule

	Semester 1	Semester 2
Year 1	Elementary Teachers Study Group Undergraduate Courses	Elementary Teachers Study Group Secondary Teachers Study Group Elementary school Follow-up Teaching Undergraduate Courses
Year 2	Secondary Teachers Study Groups Elementary Teachers Follow-up Teaching Secondary School Follow-up Teaching Undergraduate Courses	Elementary School Follow-up Teaching Secondary School Follow-up Teaching Undergraduate Courses
Year 3	Elementary School Follow-up Teaching Secondary School Follow-up Teaching Undergraduate Courses	Undergraduate Courses

(f) Outcomes of Experimental Teaching

i) Input to Research Seminar for theoretical studies and publications.
ii) Materials for taxonomy on Critical Barriers.
iii) Materials for taxonomy of teaching strategies.
iv) Case History studies from study groups and classes.
v) Conceptual material for demographic studies.

Because the specific courses we plan are organized around conceptual barriers already identified or strongly suspected, we expect that the latter will be frequently visible for documentation and analysis, and that in the sequel we will have substantial evidence of success or failure, in their diagnostic use.

C. Demographic Sampling Studies

We plan to devise a variety of techniques and instruments by which the prevalence of various specific barrier phenomena in specific sample populations can be estimated. (See discussion above). The purpose is not to achieve accurate estimates but only to obtain a rough measure of prevalence, which at least in some cases we expect on preliminary evidence to be high. For example: Given an empty test tube inverted in water (pneumatic trough) and a syringe with flexible plastic tubing attached, one is asked to fill the tube with water. Most "naive" persons will attempt to pump water into the inverted rubber tube rather than exhaust air from it, and are surprised when the water level in the tube rails to rise. This test and its opposite (to empty the full tube), is related also to conceptual difficulty with the siphon and the liquid barometer (the "ocean of air"). It has been tried only in small groups with manipulative equipment and may not prove reducible to pictorial, pencil-and-paper form. We will attempt to design ways of administering such tests which are applicable to reasonable numbers of subjects, and which permit comparison across subpopulations.

In the one statistical study of this kind that we have conducted (See Appendix A, p.1), we have found a strong majority prevalence of one critical barrier among university graduate students (30-35), elementary school teachers (20-25), and 4th grade children (25). The only exceptional group has been, predictably, graduate students from math and physics. Unless specific barrier phenomena are widespread among specific populations, they are not of the importance we ascribe to them. If they are widespread, rough frequency estimates are sufficient for our purposes, and require modest sample sizes. Our central difficulty is therefore not that of statistical reliability, but of proper sampling and of designing test situations which give unambiguous results even for small groups of subjects. This was possible in the one care referred to, and we will have expert critical help both in replicating this and extending the method to others that we judge important.

D. Investigations with Minority Groups

Our investigations of critical barrier phenomena to date have largely been based on populations from the majority culture. It seems quite possible that members of minority groups with substantially different cultural and linguistic backgrounds may approach the various topics in which we have found barriers from vantage points which would either give them a different perspective on these barriers or which might give rise to different barriers. We are well situated to explore these questions, since the Mountain View Center, with which we have been associated for almost a decade, has a long association of involvement with Blacks, Hispanics, and Native Americans -- the closest link being with the Oglalla Sioux in South Dakota and, more recently, with the Hopi in Arizona. We regard our new association with the Hopi as especially promising in the context of critical barrier research since their substantially different life outlook and the different structure of their language may well profoundly affect their common-sense perceptions of the world in which the barriers are typically encountered.

VII. Dissemination

Because of its major emphasis on the study of teaching-learning interaction the proposed research promises to provide one specific link between research and classroom innovation, and thus to create information for dissemination through·

(a) research journals such as The Journal of Research in Science Teaching and Science Education, The Journal of Educational Psychology, The Journal for Research in Mathematics Education, The European Journal of Science Education, Cognitive Science.

(b) Teachers' Journals, such as Mathematics Teacher and Mathematics Teaching, School Science and Mathematics, Science and Children, and other more general teachers' journals including OUTLOOK.

(c) Pamphlets containing illustrative case history materials and classroom-oriented discussions of conceptual development in relation to barrier pheomena and teaching materials and procedures.

(d) A small number of video tape records of teaching sessions.

A special dissemination channel for materials such as the above will be provided through the Teachers' Center Exchange of San Francisco, which maintains a network of communication with established teachers' centers and those newly created under the Teachers' Center Program of the U.S.O.E.

A further kind of dissemination, looking toward programs of informal science education, will be possible through our linkage with the San Francisco Exploratorium, and through them with Science museums and children's museums.

International dissemination of both research and teaching materials will be possible through our association with members of Her Majesty's Inspectorate in England and Wales, through the Università e Scuola group at the University of Rome, and through associates in Holland, Britain, and elsewhere.